International Journal of Scientific Research and Modern Education (IJSRME)
Impact Factor: 6.225, ISSN (Online): 2455 – 5630
(www.rdmodernresearch.com) Volume 2, Issue 1, 2017

ANTI FUZZY SOFT GAMMA REGULAR SEMIGROUPS

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Cite This Article: V. Chinnadurai & K. Arulmozhi, "Anti Fuzzy Soft Gamma Regular Semigroups", International Journal of Scientific Research and Modern Education, Volume 2, Issue 1, Page Number 159-170, 2017.

Abstract:

In this paper, we have introduce the notion of anti fuzzy soft Γ-semigroups, anti fuzzy soft Γ-ideal, bi-ideal, interior ideal Γ-regular, soft Γ-regular anti fuzzy soft Γ-regular and obtain some interesting properties results and discuss in this paper

Index Terms: Soft set, fuzzy set, Anti fuzzy soft set, soft Γ-semigroups, soft Γ-ideals (bi-ideal, interior ideal), Γ-regular semigroups, and anti fuzzy soft Γ-regular semigroups.

1. Introduction:

The fundamental concept of fuzzy set was introduced by Zadeh [16] in 1965. Sen and Saha [12] defined the Gamma semigroup in 1986. Soft set theory proposed by Molotsov [6] in 1999. Maji et al [5] worked on soft set theory and fuzzy soft set theory. Ali et al [1] introduced new operations on soft sets. Chinram and Jirojkul [3] defined the bi-ideals in Gamma semigroups in 2007. P. Dheena et al [4] studied the characterization of regular Gamma semigroups through fuzzy ideals in 2007. Chinnadurai [2] worked on regular semigroups. Characterized by the properties of anti fuzzy ideals in semigroups proposed by Shabir et al [13] anti fuzzy Gamma bi-ideal introduced in Gamma semigroups studied Nagaiah [9]. Samit Kumar Majumder studied [14] Gamma semigroups in terms of anti fuzzy ideals. Muhammad Gulistan et al [8] presented generalized anti fuzzy interior ideals in LA- semigroups. Sardar et al [11] Characterized Gamma semigroups in terms of anti fuzzy ideals. Sujit Kumar Sardar [14] worked Fuzzy Ideals in Gamma Semigroups. Muhammad Ifran Ali et al [7] studied soft ideals over semigroups. Thawhat changphas et al [15] studied soft Gamma semigroups.

In this paper, we obtained some results on properties of anti fuzzy soft Gamma semigroups and regular semigroups.

2. Preliminaries:

Definition 2.1 [12]. Let $S=\{a,b,c,\ldots\}$ and $\Gamma=\{\alpha,\beta,\gamma,\ldots\}$ be two non-empty sets. Then S is called a Gamma semigroup if it satisfies the conditions

(i) $a\alpha b\in S$

(ii) $(a\beta b)\gamma c=a\beta(b\gamma c)$ for all $a,b,c\in S$ and $\alpha,\beta,\gamma\in\Gamma$.

Definition 2.2 [15]. A $\Gamma-$semigroup S is called a regular if for each element $a\in S$, there exists $x\in S$ and $\alpha,\beta\in\Gamma$ such that $a=a\alpha x\beta a$.

Definition 2.3 [4]. Let S be a $\Gamma-$ semigroup. A non empty subset A of S is called an ideal, an ideal A of S is said to be idempotent A Γ A = A.

Definition 2.4 [6]. Let U be the universel set, E be the set of parameters, $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F,A) is called a soft set over U, where F is mapping given by $F:A\to P(U)$.

Definition 2.5 [5]. Let (F,A) and (G,B) be two soft sets over a common universe U then (F,A) AND (G,B) denoted by $(F,A)\wedge(G,B)$ is defined as $(F,A)\wedge(G,B)=(H,A\times B)$ where $H(\alpha,\beta)=F(\alpha)\cap G(\beta),\ \forall(\alpha,\beta)\in A\times B$

Definition 2.6 [5]. Let (F,A) and (G,B) be two soft sets over a common universe U then (F,A) OR (G,B) denoted by $(F,A)\vee(G,B)$ is defined as $(F,A)\vee(G,B)=(H,A\times B)$ where $H(\alpha,\beta)=F(\alpha)\cup G(\beta)\ \forall(\alpha,\beta)\in A\times B.$

Definition 2.7 [1]. The extended union of two fuzzy soft sets (F,A) and (G,B) over a common universe U is fuzzy soft set denoted by $(F,A)\cup_{\in}(G,B)$ defined as $(F,A)\cup_{\in}(G,B)=(H,C)$ where

$C = A \cup B, \ \forall c \in C.$

$$H(c) = \begin{cases} F(c) & if \ c \in A - B \\ G(c) & if \ c \in B - A \\ F(c) \cup G(c) & if \ c \in A \cap B. \end{cases}$$

Definition 2.8 [1]. The extended intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is fuzzy soft set denoted by $(F, A) \cap_{\in} (G, B)$ defined as $(F, A) \cap_{\in} (G, B) = (H, C)$ where $C = A \cup B, \ \forall c \in C.$

$$H(c) = \begin{cases} F(c) & if \ c \in A - B \\ G(c) & if \ c \in B - A \\ F(c) \cap G(c) & if \ c \in A \cap B. \end{cases}$$

Definition 2.9 [2]. A soft set (F, A) is called a soft semigroup over S if, $(F, A) \tilde{\circ} (F, A) \subseteq (F, A)$. Clearly a soft set (F, A) over a semigroup S is a soft semigroup if and only if $\phi \neq F(a)$ is a subsemigroup of S, $\forall a \in A.$

Definition 2.10 [7]. A soft semigroup (F, A) over a semigroup S is called a soft regular semigroup if for each $\alpha \in A, F(\alpha)$ is regular.

Definition 2.11 [2]. The restricted product (H, C) of two fuzzy soft sets (F, A) and (G, B) over a semigroup S is defined as $(H, C) = (F, A) \tilde{\circ} (G, B)$ where $C = A \cap B$ by $H(c) = F(c) \tilde{\circ} G(c), \forall c \in C.$

Definition 2.12 [16]. Let X be non-empty set. A fuzzy subset μ of X is a function from X into the closed unit interval [0, 1]. The set of all fuzzy subsets of X is called a fuzzy power set of X and is denoted by $FP(X)$.

Definition 2.13 [8]. Let X be non empty set and A be subset of X. Then the anti characterization function $\chi_A{}^c$ is defined by soft $\Gamma -$semigroup of S.

$$\chi_A{}^c = \begin{cases} 0 \ if & x \in A \\ 1 \ if & x \notin A \end{cases}$$

Definition 2.14 [11]. A fuzzy set δ of a $\Gamma -$semigroup S is called an anti fuzzy $\Gamma -$subsemigroup of S if $\delta(x \alpha y) \leq \max\{\delta(x), \delta(y)\}$ for all $x, y \in S$ and $\alpha \in \Gamma$.

Definition 2.15 [11]. A fuzzy set δ of a $\Gamma -$semigroup S is called an anti fuzzy $\Gamma -$ left (right) ideal of S if $\delta(x \alpha y) \leq \delta(y)$ $(\delta(x \alpha y) \leq \delta(x))$ for all $x, y \in S$ and $\alpha \in \Gamma$.

Definition 2.16 [13]. A fuzzy set δ of a $\Gamma -$semigroup S is called an anti fuzzy $\Gamma -$bi-ideal of S if $\delta(x \alpha y \beta z) \leq \max\{\delta(x), \delta(z)\}$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

Definition 2.17 [13]. A fuzzy set δ of a $\Gamma -$semigroup S is called an anti fuzzy $\Gamma -$interior ideal of S if $\delta(x \alpha y \beta z) \leq \delta(y)$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

Definition 2.18 [10]. Let δ be a fuzzy subset of a $\Gamma -$semigroup S and let $t \in [0,1]$ then the set $\delta_t = \{x \in S : \delta(x) \leq t\}$ is called the anti level subset of δ.

3. Anti Fuzzy Soft $\Gamma -$Semigroups:

In this section S denotes anti fuzzy soft $\Gamma -$semigroup and AFS denotes anti fuzzy soft.

Definition 3.1. Let (F_1, A) be AFS set over a $\Gamma -$semigroup S, then $(F_1, A) \tilde{\circ} (F_1, A) \supseteq (F_1, A)$ is called AFS $\Gamma -$semigroup.

Example 3.2. $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ where α, β is defined on S with the following Cayley table:

α	a_1	a_2	a_3
a_1	a_1	a_1	a_1
a_2	a_1	a_1	a_2
a_3	a_1	a_1	a_3

β	a_1	a_2	a_3
a_1	a_1	a_1	a_1
a_2	a_1	a_1	a_1
a_3	a_1	a_1	a_1

Table-1

Consider $A=\{a_1,a_2,a_3\}$ and $F(a_1)=\{a_1\}$, $F(a_2)=\{a_1,a_2\}$, $F(a_3)=\{a_1,a_3\}$.

Hence (F_1,A) is soft $\Gamma-$semigroup.

Definition 3.3. Let (F_1,A) be AFS set over a $\Gamma-$semigroup S, then (F_1,A) is called AFS $\Gamma-$subsemigroup of S if, $\delta_e(p\alpha q)\le\max\{\delta_e(p),\delta_e(q)\}$ for all $p,q\in S$ and $\alpha\in\Gamma$.

Definition 3.4 Let (F_1,A) be AFS set over a $\Gamma-$subsemigroup S then (F_1,A) is called AFS $\Gamma-$left (right)deal of S if $\delta_e(p\alpha q)\le\delta_e(q)$ $(\delta_e(p\alpha q)\le\delta_e(p))$ for all $p,q\in S$ and $\alpha\in\Gamma$.

Definition 3.5. Let (F_1,A) be AFS set over a $\Gamma-$semigroup S, then (F_1,A) is called AFS $\Gamma-$ideal of S if, $\delta_e(p\alpha q)\le\min\{\delta_e(p),\delta_e(q)\}$ for all $p,q\in S$ and $\alpha\in\Gamma$.

Definition 3.6. Let (F_1,A) be AFS set over a $\Gamma-$semigroup S, then (F_1,A) is called AFS $\Gamma-$bi-ideal of S if, $\delta_e(p\alpha r\beta q)\le\max\{\delta_e(p),\delta_e(q)\}$ for all $p,q,r\in S$ and $\alpha,\beta\in\Gamma$.

Definition 3.7. Let (F_1,A) be AFS set over a $\Gamma-$semigroup S, then (F_1,A) is called AFS $\Gamma-$interior ideal of S if, $\delta_e(p\alpha r\beta q)\le\delta_e(r)$ for all $p,q,r\in S$ and $\alpha,\beta\in\Gamma$.

Theorem3.8. Let (F_1,A) be AFS subset of S. If (F_1,A) be AFS subsemigroup of S if and only if $(F_1,A)\tilde{\circ}(F_1,A)\supseteq(F_1,A)$.

Proof. Let $u\in S$. Assume that (F_1,A) is AFS $\Gamma-$subsemigroup of S

$(F_1,A)\tilde{\circ}(F_1,A)=(H_1,A)$. where $H_1(e)=F_1(e)\tilde{\circ}F_1(e)$ for all $e\in A$.

Consider

$$\begin{aligned}(F_1,A)\tilde{\circ}(F_1,A)(u)&=\inf_{u=a\Gamma b}\max\{F_1(e)(a),F_1(b)\}\\&\ge\inf_{u=a\Gamma b}F_1(e)(a\Gamma b)\\&\ge\inf_{u=a\Gamma b}F_1(e)(u)\\&=F_1(e)(u)\end{aligned}$$

$(F_1,A)\tilde{\circ}(F_1,A)\supseteq(F_1,A)$.

Conversely assume that $(F_1,A)\tilde{\circ}(F_1,A)\supseteq(F_1,A)$ $\forall u,v\in S$

Consider

$$\begin{aligned}(F_1(e)(u\Gamma v)&\le\{F_1(e)\tilde{\circ}F_1(e)\}(u\Gamma v)\\&=\inf_{u\Gamma v=a\Gamma b}\max\{F_1(e)(a),F_1(b)\}\\&\le\max\{F_1(e)(u),F_1(e)(v)\}\end{aligned}$$

Hence (F_1,A) be AFS subset of a $\Gamma-$subsemigroup of S

Theorem 3.9. Let (F_1,A) be AFS subset of a $\Gamma-$semigroup S. Then the following conditions hold.

(i) (F_1,A) be AFS $\Gamma-$left ideal of S if and only if $(S,E)\tilde{\circ}(F_1,A)\supseteq(F_1,A)$.

(ii) (F_1,A) be AFS $\Gamma-$right ideal of S if and only if $(F_1,A)\tilde{\circ}(S,E)\supseteq(F_1,A)$.

Proof. We prove that (i) holds. Assume that (F_1,A) be AFS $\Gamma-$left ideal of S $(S,E)\tilde{\circ}(F_1,A)=(H_1,A)$ where $H_1(e)=S(e)\tilde{\circ}F_1(e)$ $\forall e\in A$.

Consider

$$\begin{aligned}((S,E)\tilde{\circ}(F_1,A))(u) &= \inf_{u=a\Gamma b} \max\{S(e)(a), F_1(e)(b)\} \\ &\geq \inf_{u=a\Gamma b} \max\{0,\ F_1(e)(a\Gamma b)\} \\ &= \max\{0, F_1(e)(u)\} \\ &= F_1(e)(u)\end{aligned}$$

So $(S,E)\tilde{\circ}(F_1,A) \supseteq (F_1,A)$.

Conversely assume that $(S,E)\tilde{\circ}(F_1,A) \supseteq (F_1,A)$.

Let $u = a\Gamma b$ then we have

$$\begin{aligned}F_1(e)(a\Gamma b) &= F_1(e)(u) \\ &\leq \{S(e)\tilde{\circ}F_1(e)\}(u) \\ &= \inf_{u=p\Gamma q} \max\{S(e)(p), F_1(e)(q)\} \\ &\leq \max\{S(e)(a), F_1(e)(b)\} \\ &= \max\{0,\ F_1(e)(b)\} \\ &= F_1(e)(b)\end{aligned}$$

Hence (F_1,A) is AFS $\Gamma-$left ideal of S

(ii) Similar proof.

Theorem 3.10. Let (F_1,A) be AFS $\Gamma-$subsemigroup of S . Then (F_1,A) is AFS $\Gamma-$bi-ideal of S if and only if $(F_1,A)\tilde{\circ}(S,E)\tilde{\circ}(F_1,A) \supseteq (F_1,A)$.

Proof. Let (F_1,A) is AFS $\Gamma-$bi-ideal of S, then

$(F_1,A)\tilde{\circ}(S,E)\tilde{\circ}(F_1,A) = (H_1,A)$ where $H_1(e) = F_1(e)\tilde{\circ}S(e)\tilde{\circ}F_1(e)$ for all $e \in A$.

Let $u \in S$ such that $u = a\Gamma b$ and $a = p\Gamma q$.

We have $F_1(e)(p\Gamma q\Gamma b) \leq \max\{F_1(e)(p), F_1(e)(b)\}$

Consider

$$\begin{aligned}F_1((e)\tilde{\circ}S(e)\tilde{\circ}F_1(e))(u) &= \inf_{u=a\Gamma b}\{\max\{(F_1(e)\tilde{\circ}S(e))(a), F_1(e)(b)\}\} \\ &\geq \inf_{u=a\Gamma b}\left\{\max\left\{\inf_{a=p\Gamma q}\{\max\{F_1(e)(p), S(e)(q)\}\}F_1(e)(b)\right\}\right\} \\ &= \inf_{u=a\Gamma b}\left\{\max\left\{\inf_{a=p\Gamma q}\{\max\{F_1(e)(q), 0\}\}F_1(e)(b)\right\}\right\} \\ &= \inf_{u=p\Gamma q\Gamma b}\{\max\{F_1(e)(p), F_1(e)(b)\}\} \\ &\geq \inf_{u=p\Gamma q\Gamma b} F_1(e)(p\Gamma q\Gamma b) \\ &= F_1(e)(u)\end{aligned}$$

Therefore $F_1(e)\tilde{\circ}S(e)\tilde{\circ}F_1(e) \supseteq F_1(e)$ for all $e \in A$.

Conversely assume that $F_1(e)\tilde{\circ}S(e)\tilde{\circ}F_1(e) \supseteq F_1(e)$

Let $a,b,c \in S$ and $u = a\Gamma b\Gamma c$, we have

$$F_1(e)(a\Gamma b\Gamma c) = F_1(e)(u)$$

$$\leq \{F_1(e)\tilde{\circ}S(e)\tilde{\circ}F_1(e)\}(u)$$

$$= \inf_{u-x\Gamma y}\{\max\{F_1(e)\tilde{\circ}S(e)\}(x), F_1(e)(y)\}$$

$$\leq \max\{\{F_1(e)\tilde{\circ}S(e)\}(a\Gamma b), F_1(e)(c)\}$$

$$= \max\left\{\inf_{a\Gamma b=p\Gamma q}\{\max\{F_1(e)(p), S(e)(q)\}\}, F_1(e)(c)\right\}$$

$$\leq \max\{\max\{F_1(e)(a), S(e)(b)\}, F_1(e)(c)\}$$

$$= \max\{\max\{F_1(e)(a), 0\}F_1(e)(c)\}$$

$$= \max\{F_1(e)(a), F_1(e)(c)\}$$

Hence (F_1, A) is AFS $\Gamma-$bi-ideal of S.

Theorem 3.11. Let (F_1, A) AFS be subset of a $\Gamma-$semigroup of S . If (F_1, A) is AFS $\Gamma-$bi-ideal of S then $(F_1, A)\tilde{\circ}(F_1, A) \supseteq (F_1, A)$ and $(F_1, A)\tilde{\circ}(S, E)\tilde{\circ}(F_1, A) \supseteq (F_1, A)$.

Proof. By theorem (3.8) and (3.10) we get the required results.

Theorem 3.12. Let (F_1, A) be AFS $\Gamma-$subset of a $\Gamma-$ Semigroup S . Then (F_1, A) is AFS soft $\Gamma-$interior ideal of S if and only if $(S,E)\tilde{\circ}(F_1, A)\tilde{\circ}(S,E) \supseteq (F_1, A)$.

Proof

Assume that (F_1, A) is AFS $\Gamma-$interior ideal of S, now

$(S,E)\tilde{\circ}(F_1, A)\tilde{\circ}(S,E) = (H_1, A)$ where $H_1(e) = S(e)\tilde{\circ}F_1(e)\circ S(e)$ for all $e \in A$.

Let $u \in S$ such that $u = a\Gamma b$ and $a = p\Gamma q$.

We have $F_1(e)(p\Gamma q\Gamma b) \leq F_1(e)(q)$

Consider

$$(S(e)\tilde{\circ}F_1(e)\tilde{\circ}S(e))(u) = \inf_{u=a\Gamma b}\{\max\{\{S(e)\tilde{\circ}F_1(e)\}(a), S(e)(b)\}\}$$

$$= \inf_{u=a\Gamma b}\left\{\max\left\{\inf_{a=p\Gamma q}\{\max\{S(e)(p), F_1(e)(q)\}\}S(e)(b)\right\}\right\}$$

$$\geq \inf_{u=a\Gamma b}\left\{\max\left\{\inf_{a=p\Gamma q}\{\max\{0, F_1(e)(q)\}\}, 0\right\}\right\}$$

$$\geq F_1(e)(u)$$

Therefore $S(e)\tilde{\circ}F_1(e)\tilde{\circ}S(e) \supseteq F_1(e)$

Conversely assume that $S(e)\tilde{\circ}F_1(e)\tilde{\circ}S(e) \supseteq F_1(e)$

Let $u, w, v \in S$

$$F_1(e)(u\Gamma w\Gamma v) \leq \{S(e)\tilde{\circ} F_1(e)\tilde{\circ} S(e)\}(u\Gamma w\Gamma v)$$

$$= \inf_{u\Gamma w\Gamma v = p\Gamma q} \{\max\{S(e)\tilde{\circ} F_1(e)\}(p), S(e)(q)\}$$

$$\leq \max\{(S(e)\tilde{\circ} F_1(e))(u\Gamma w), S(e)(v)\}$$

$$= \max\{(S(e)\tilde{\circ} F_1(e))(u\Gamma w), 0\}$$

$$= \inf_{u\Gamma w = p\Gamma q} \{\max\{S(e)(p), F_1(e)(q)\}\}$$

$$\leq \max\{S(e)(u), F_1(e)(w)\}$$

$$= \max\{0, F_1(e)(w)\}$$

$$= F_1(e)(w).$$

Hence (F_1, A) is AFS $\Gamma-$interior ideal of S.

Theorem 3.13 Let (F_1, A) be AFS subset of S. Then (F_1, A) is AFS subsemigroup of S if and only if $\tilde{U}((F_1, A):[t_1, t_2])$ is AFS subsemigroup of S.

Proof. Assume that (F_1, A) is AFS subset of S. Let $[t_1, t_2] \in [0,1]$ such that $p, q \in \tilde{U}((F_1, A):[t_1, t_2])$ then

$$\begin{aligned} \delta_{F_1(e)}(p\alpha q) &\leq \max\{\delta_{F_1(e)}(p), \delta_{F_1(e)}(q)\} \\ &\leq \max\{[t_1, t_2], [t_1, t_2]\} \\ &= [t_1, t_2] \end{aligned}$$

Thus $pq \in \tilde{U}((F_1, A):[t_1, t_2])$. Hence $\tilde{U}((F_1, A):[t_1, t_2])$ is AFS $\Gamma-$subsemigroup over S. Conversely, assume that $\tilde{U}((F_1, A):[t_1, t_2])$ is AFS $\Gamma-$subsemigroup of S, for all $[t_1, t_2] \in [0,1]$ and $p, q \in S, \alpha \in \Gamma$. Suppose $\delta_{F_1(e)}(p\alpha q) > \max\{\delta_{F_1(e)}(p), \delta_{F_1(e)}(q)\}$ then there exists an element $x \in [0, 1]$ such that $\delta_{F_1(e)}(p\alpha q) > x > \max\{\delta_{F_1(e)}(p), \delta_{F_1(e)}(q)\}$, which implies that $\delta_{F_1(e)}(p) > x$ and $\delta_{F_1(e)}(q) > x$ then we have $p, q \in \tilde{U}((F_1, A): x)$. Hence $\delta_{F_1(e)}(pq) < x$ which is a contradiction, then we have $p, q \in \tilde{U}((F_1, A): x)$.

Theorem 3.14. Let (F_1, A) and (G_1, B) be two AFS $\Gamma-$ideal (bi-ideal, interior ideal) of S, then $(F_1, A) \wedge (G_1, B)$ is AFS $\Gamma-$ideal (bi-ideal, interior ideal) of S.

Proof. Let (F_1, A) and (G_1, B) be two AFS $\Gamma-$ideal over S. Now we defined $(F_1, A) \wedge (G_1, B) = (H_1, C)$ where $C = A \times B$ and $H_1(a,b) = F_1(a) \cap G_1(b)$ for all $(a,b) \in C.$

$$\begin{aligned}\delta_{H_1(a,b)}(l\alpha m) &= \left(\delta_{F_1(a)} \cap \delta_{G_1(b)}\right)(l\alpha m)\\ &= \max\left\{\delta_{F_1(a)}(l\alpha m), \delta_{G_1(b)}(l\alpha m)\right\}\\ &\le \max\left\{\min\left\{\delta_{F_1(a)}(l), \delta_{F_1(a)}(m)\right\}, \min\left\{\delta_{G_1(b)}(l), \delta_{F_1(b)}(m)\right\}\right\}\\ &= \min\left\{\max\left\{\delta_{F_1(a)}(l), \delta_{G_1(b)}(l)\right\}, \max\left\{\delta_{F_1(a)}(m), \delta_{G_1(b)}(m)\right\}\right\}\\ &= \min\left\{\left(\delta_{F_1(a)} \cap \delta_{G_1(b)}\right)(l), \left(\delta_{F_1(a)} \cap \delta_{G_1(b)}\right)(m)\right\}\\ &= \min\left\{\delta_{H_1(a,b)}(l), \delta_{H_1(a,b)}(m)\right\}\end{aligned}$$

Hence $(F_1, A) \wedge (G_1, B)$ is AFS $\Gamma-$ideal of S.

Theorem 3.15. Let (F_1, A) and (G_1, B) be two AFS $\Gamma-$ideal (bi-ideal, interior ideal) of S, then $(F_1, A) \vee (G_1, B)$ is AFS $\Gamma-$ideal (bi-ideal, interior ideal) of S.

Proof. The proof is straightforward.

Example 3.16. Every AFS $\Gamma-$ideal of S is an AFS $\Gamma-$bi-ideal of S, but converse is not true. From the table.1, Let $E = \{v_1, v_2, v_3\}$, $A = \{v_1, v_3\}$, then (F_1, A) is AFS set defined as

$$\delta_{F_1(v_1)} = \{(a_1, 0.2), (a_2, 0.8), (a_3, 0.5)\},$$

$$\delta_{F_1(v_3)} = \{(a_1, 0.4), (a_2, 0.9), (a_3, 0.7)\}$$

Hence (F_1, A) is AFS $\Gamma-$bi-ideal , but not $\Gamma-$ideal of S.

Theorem 3.17. Let (F_1, A) and (G_1, B) be two AFS $\Gamma-$ideal of S, then $(F_1, A) \cap_{\in} (G_1, B)$ and $(F_1, A) \cup_{\in} (G_1, B)$ is AFS $\Gamma-$ideal of S.

Proof. Let (F_1, A) and (G_1, B) be two AFS $\Gamma-$ideal of S then $(F_1, A) \cap (G_1, B) = (H, C)$ where $C = A \times B,$

$$H_1(c) = \begin{cases} F_1(c) & if \;\; c \in A - B \\ G_1(c) & if \;\; c \in B - A \\ F_1(c) \cap G_1(c) & if \;\; c \in A \cap B \end{cases}$$

Let $p, q \in S$ and $\alpha \in \Gamma$

(i) If $c \in A - B$

$$\begin{aligned}\delta_{H_1(c)}(p\alpha q) &= \delta_{F_1(c)}(l\alpha m)\\ &\le \min\left\{\delta_{F_1(c)}(l), \delta_{F_1(c)}(m)\right\}\\ &= \min\left\{\delta_{H_1(c)}(l), \delta_{H_1(c)}(m)\right\}\end{aligned}$$

(ii) $c \in B - A$

$$\begin{aligned}\delta_{H_1(c)}(l\alpha m) &= \delta_{G_1(c)}(p\alpha q)\\ &\le \min\left\{\delta_{G_1(c)}(p), \delta_{G_1(c)}(q)\right\}\\ &= \min\left\{\delta_{H_1(c)}(p), \delta_{H_1(c)}(q)\right\}\end{aligned}$$

(iii) If $c \in A \cap B$ then $H_1(c) = \max\{F_1(c), G_1(c)\} = \{F_1(c) \cap G_1(c)\}.$

Now using seen that $H_1(c)(l\alpha m) \le \min\{H_1(c)(l), H_1(c)(m)\}$ for all $l, m \in S$ $, \alpha \in \Gamma$ and $c \in C$

$$\delta_{H_1(c)}(l\alpha m) \le \min\left\{\delta_{H_1(c)}(l), \delta_{H_1(c)}(m)\right\}.$$

Hence $(F_1, A) \cap_{\in} (G_1, B)$ is AFS $\Gamma-$ ideal over S

Theorem 3.18. Let (F_1, A) and (G_1, B) be two AFS $\Gamma-$ideal of S, then $(F_1, A) \cap_{\in} (G_1, B)$ and $(F_1, A) \cup_{\in} (G_1, B)$ is AFS $\Gamma-$bi-ideal of S.

Proof. The proof is straightforward.

4. Anti fuzzy soft gamma regular semigroups.

In this section S denotes the soft $\Gamma -$regular semigroup.

Definition 4.1. A soft $\Gamma -$semigroup (F_1, A) over a semigroup S is called a soft $\Gamma -$regular semigroup if for each $\alpha, \beta \in A, \ F_1(\alpha, \beta)$ is regular.

Example 4.2. $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ where α, β is defined on S with the following Cayley table:

α	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_3	a_3	a_3

β	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

Table-2

Consider $E = \{a_1, a_2, a_3, a_4\}$ and $F(a_1) = \{a_1, a_3\}$, $F(a_2) = \{a_2, a_3\}$, $F(a_3) = \{a_1, a_2, a_3\}$ $F(a_4) = \{a_2, a_3, a_4\}$. Hence (F, S) is soft $\Gamma -$regular semigroup.

Theorem 4.3. Let (F_1, A) and (G_1, B) be two AFS sets of soft $\Gamma -$regular semigroup S and A_1 and A_2 are two non-empty subsets of S.

$(i)\ \chi^c_{A_1} \tilde{\cap} \chi^c_{A_2} = \chi^c_{A_1 \cap A_2}$

$(ii)\ \chi^c_{A_1} \tilde{\circ} \chi^c_{A_2} = \chi^c_{A_1 \Gamma A_2}$

Proof. Let $p \in A_1 \cap A_2$, then $p \in A_1$ and $p \in A_2$. we have

$$\left(\chi^c_{A_1} \tilde{\cap} \chi^c_{A_2}\right)(p) = \max\{\chi^c_{A_1}(p),\ \chi^c_{A_1}(p)\}$$
$$= \max\{0,0\}$$
$$= 0$$
$$= \chi^c_{A_1 \cap A_2}(p)$$

Suppose $p \notin A_1 \cap A_2$, then $p \notin A_1$ and $p \notin A_2$

$$\left(\chi^c_{A_1} \tilde{\cap} \chi^c_{A_2}\right)(p) = \max\{\chi^c_{A_1}(p),\ \chi^c_{A_1}(p)\}$$
$$= \max\{1,1\}$$
$$= 1$$
$$= \chi^c_{A_1 \cap A_2}(p)$$

(ii) Let $p \in S$, suppose $p \in A_1 \Gamma A_2$, then there exists $a_1 \in A_1,\ \gamma \in \Gamma$ and $a_2 \in A_2$,such that $p = a_1 \gamma a_2$

$$\left(\chi^c_{A_1} \tilde{\circ} \chi^c_{A_2}\right)(p) = \inf_{p = c\gamma d} \max\{\chi^c_{A_1}(c) \tilde{\circ} \chi^c_{A_2}(d)\}$$
$$\leq \max\left\{\chi^c_{A_1}(p)\, \chi^c_{A_2}(p)\right\}$$
$$= \max\{0,\ 0\}$$
$$= 0$$

Since $1 \geq \left(\chi^c_{A_1} \tilde{\circ} \chi^c_{A_2}\right)(p) \geq 0,$ hence $\chi^c_{A_1} \tilde{\circ} \chi^c_{A_2}(p) = 0 = \chi^c_{A_1 \Gamma A_2}$

Suppose $p \notin A_1 \Gamma A_2$, then $p \notin a_1 \gamma a_2, a_1 \in A_1, \gamma \in \Gamma$ and $a_2 \in A_2$.

$$\left(\chi^c_{A_1} \tilde{\circ} \chi^c_{A_2}\right)(p) = \max\{\chi^c_{A_1}(p), \ \chi^c_{A_1}(p)\}$$
$$= \max\{1,1\}$$
$$= 1$$
$$= \chi^c_{A_1 \Gamma A_2}(p)$$

Hence $\chi^c_{A_1} \tilde{\circ} \chi^c_{A_2} = \chi_{A_1 \Gamma A_2}$

The following theorem relation between soft $\Gamma-$semigroup and AFS $\Gamma-$semigroups.

Theorem 4.4. Let (F_1, A) be a non-empty soft subset of S, (F_1, A) be a soft $\Gamma-$ subsemigroup of S if and only if $\chi^c_{F_1(e)}$ is AFS $\Gamma-$subsemigroup of S.

Proof. Let (F_1, A) be a soft $\Gamma-$ semigroup of S

$$\chi^c_{F_1(e)} \lambda(e) = \begin{cases} 0 & if \ \ a \in F_1(e) \\ 1 & if \ \ a \notin F_1(e) \end{cases}$$

Let $a, b \in S, \gamma \in \Gamma$ $\chi^c_{F_1(e)} \lambda(a\gamma b) \geq \max\left\{\chi^c_{F_1(e)} \lambda(a), \chi^c_{F_1(e)} \lambda(b)\right\}$

then $\chi^c_{F_1(e)} \lambda(a) = 0,$ $\chi^c_{F_1(e)} \lambda(b) = 0,$ and $\chi^c_{F_1(e)} \lambda(a\gamma b) = 1,$ this implies that $a, b \in F_1(e)$ since $F_1(e)$ is a $\Gamma-$subsemigroup of S, $a\gamma b \in F_1(e)$ and hence $\chi^c_{F_1(e)} \lambda(a\gamma b) = 0$ which is a contradiction.

Thus $\chi^c_{F_1(e)} \lambda(a\gamma b) \geq \max\left\{\chi^c_{F_1(e)} \lambda(a), \chi^c_{F_1(e)} \lambda(b)\right\}$ for all $a, b \in F_1(e)$ and $e \in A.$

Conversely assume that $\chi^c_{F_1(e)}$ is AFS $\Gamma-$subsemigroup of S. Let $a, b \in F_1(e)$ then $\chi^c_{F_1(e)} \lambda(a) = 0$ and $\chi^c_{F_1(e)} \lambda(b) = 0$, $\chi^c_{F_1(e)}$ AFS $\Gamma-$subsemigroup.

Now $\max\left\{\chi^c_{F_1(e)} \lambda(a), \chi^c_{F_1(e)} \lambda(b)\right\} == 0 \geq \chi^c_{F_1(e)} \lambda(a\gamma b)$ this implies that $\chi^c_{F_1(e)} \lambda(a\gamma b) = 0$ and hence $a, b \in F_1(e)$ for all $e \in A$ .Therefore (F_1, A) is a soft $\Gamma-$subsemigroup of S

Theorem 4.5. Let (F_1, A) be a soft $\Gamma-$ideal of S if and only if $\chi^c_{F_{1A}}$ (characteristic function) is AFS $\Gamma-$ ideal of soft $\Gamma-$regular semigroup S.

Theorem 4.6. Let (F_1, B) be a soft $\Gamma-$bi-ideal of S if and only if $\chi^c_{F_{1B}}$ (characteristic function) is AFS $\Gamma-$bi-ideal of soft $\Gamma-$regular semigroup S.

Theorem 4.7.The following conditions are equivalent

(i) Every soft Γ -bi-ideal is a soft Γ -ideal of S.

(ii) Every AFS Γ -bi-ideal of S is a AFS Γ -ideal of S.

Proof. Assume that condition (i) holds, let $\delta_{F_1(e)}$ be any AFS Γ -ideal of S. Let $\delta_{F_1(e)}$ be any AFS Γ -bi-ideal of S, and $p, q \in S,$ since the set $(F_1, A) \tilde{\circ} (S, E) \tilde{\circ} (F_1, A)$ is a soft Γ -bi-ideal of S, by the assumption is soft Γ -right ideal of S, since S is a soft Γ -regular ,we have $p\Gamma q \in (p\alpha F(\alpha, \beta)\beta p)\Gamma q \subseteq p\alpha F_1(\alpha, \beta)\beta p,$ there exists $x \in S$ such that $p\Gamma q = p\alpha x\beta p,$ since $\delta_{F_1(e)}$ is AFS Γ -bi–ideal of S, for all $e \in A$

Consider

$$\begin{aligned}\delta_{F_1(e)}(p\Gamma q) &= \delta_{F_1(e)}(p\alpha x\beta p)\\ &\le \max(\delta_{F_1(e)}(p), \delta_{F_1(e)}(p))\\ &= \delta_{F_1(e)}(p)\end{aligned}$$

Hence $\delta_{F_1(e)}$ is AFS Γ-right ideal of S. Similarly $\delta_{F_1(e)}$ is AFS Γ-left ideal of S. Therefore $\delta_{F_1(e)}$ is AFS Γ-ideal of S.

Conversely assume that AFS Γ-bi-ideal of S holds. Let (F_1, B) be soft Γ-ideal of S by theorem (4.6) the characteristic function $\chi^c_{F_{1B}}$ is a AFS Γ-bi-ideal of S. Hence by assumption $\chi^c_{F_{1A}}$ is AFS Γ-ideal of S, thus by theorem (4.5) $\chi^c_{F_{1A}}$ is soft Γ-ideal of S.

Hence $(ii) \Rightarrow (i)$

The following examples shows that AFS Γ-ideal and AFS Γ-bi-ideal of S.

Examples 4.8. Let $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ then S is a Γ-semigroup under the operation defined as in the table.1.

Let $E = \{v_1, v_2, v_3, v_4\}$, $A = \{v_1, v_3\}$ then (F_1, A) is AFS set defined as,

$$\delta_{F_1(v_1)} = \{(a_1, 0.1), (a_2, 0.9), (a_3, 0.3), (a_4, 0.5)\}$$

$$\delta_{F_1(v_3)} = \{(a_1, 0.2), (a_2, 1), (a_3, 0.5), (a_4, 0.7)\}$$

Hence (F_1, A) is AFS Γ-bi-ideal and AFS Γ-ideal over S.

Theorem 4.9. In a soft Γ-regular semigroup S, every AFS Γ-two sided ideal is idempotent.

Proof. Let (F_1, A) be AFS Γ-two sided ideal of S, then by theorem (3.8) $(F_1, A)\,\tilde{\circ}\,(F_1, A) \supseteq (F_1, A)$. Since S is soft Γ-regular, $p \in S$, there exists $x \in S$, such that $p = p\alpha x\beta p$ for all $e \in A$ we have

$$\begin{aligned}(F_1(e)\,\tilde{\circ}\,F_1(e))(p) &= \inf_{p=p\alpha x\beta p} \max\{F_1(e)(p\alpha x), F_1(e)(p)\}\\ &\le \max\{F_1(e)(p\alpha x), F_1(e)(p)\}\\ &\le \max\{F_1(e)(p), F_1(e)(p)\}\\ &= F_1(e)(p)\end{aligned}$$

Therefore $(F_1, A)\,\tilde{\circ}\,(F_1, A) \subseteq (F_1, A)$.

Hence $(F_1, A)\,\tilde{\circ}\,(F_1, A) = (F_1, A)$.

Theorem 4.10. In a soft Γ-regular semigroup S, every AFS Γ-interior ideal is idempotent.

Proof. Let (F_1, A) be AFS Γ-interior ideal of S, then by theorem (3.10) $(F_1, A)\,\tilde{\circ}\,(F_1, A) \supseteq (F_1, A)$. Since S is soft Γ-regular, $p \in S$, there exists $x \in S$, such that $p = p\alpha x\beta p$ for all $e \in A$

we have $p = p\alpha x\beta p = p\alpha x\beta p\alpha x\beta p = ((p\alpha x)\beta p\alpha x)\beta p$ for all $e \in A$.

$$\begin{aligned}(F_1(e)\,\tilde{\circ}\,F_1(e))(p) &= \inf_{p=((p\alpha x)\beta p\alpha x)\beta p} \max\{F_1(e)((p\alpha x)\beta p\alpha x), F_1(e)(p)\}\\ &\le \max\{F_1(e)((p\alpha x)\beta p\alpha x), F_1(e)(p)\}\\ &\le \max\{F_1(e)(p), F_1(e)(p)\}\\ &= F_1(e)(p)\end{aligned}$$

Therefore $(F_1, A)\,\tilde{\circ}\,(F_1, A) \subseteq (F_1, A)$. Hence $(F_1, A)\,\tilde{\circ}\,(F_1, A) = (F_1, A)$.

Theorem 4.11. Every AFS $\Gamma-$ interior ideal is AFS $\Gamma-$ ideal over soft $\Gamma-$. regular semigroup S.

Proof. Let $\delta_{F_1(e)}$ be a soft $\Gamma -$.ideal of S for all $e \in A$.

We have

$$\delta_{F_1(e)}(p\alpha q\beta r) \le \delta_{F_1(e)}(q\beta r) \text{ since } \delta_{F_1(e)} \text{ is a } \Gamma-\text{left ideal of S}$$

$$\le \delta_{F_1(e)}(q) \text{ since } \delta_{F_1(e)} \text{ is a } \Gamma-\text{right ideal of S}$$

Hence $\delta_{F_1(e)}(p\alpha q\beta r) \le \delta_{F_1(e)}(q)$ for all $p, q, r \in S$ and $\alpha, \beta \in \Gamma$.

Conversely assume that $\delta_{F_1(e)}$ is AFS $\Gamma -$interior ideal of S. Let $p, q \in S$ since S is a soft $\Gamma -$regular semigroups, there exists $x, y \in S$ such that $p = p\alpha x\beta p$ and $q = q\alpha y\beta q$ and $\alpha, \beta \in \Gamma$.

Thus we have

$$\begin{aligned}\delta_{F_1(e)}(p\Gamma q) &\le \delta_{F_1(e)}((p\alpha x\beta p)\Gamma q)\\ &= \delta_{F_1(e)}((p\alpha x)\beta p\Gamma q)\\ &\le \delta_{F_1(e)}(p)\end{aligned}$$

And

$$\begin{aligned}\delta_{F_1(e)}(p\Gamma q) &\le .\delta_{F_1(e)}((p\Gamma)q\alpha y\beta q)\\ &= \delta_{F_1(e)}((p\Gamma q\alpha)y\beta q)\\ &\le \delta_{F_1(e)}(q)\end{aligned}$$

Hence proved.

Examples 4.12. Let $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ then S is a Γ-semigroup under the operation defined as in the table.1.

Let $E = \{u_1, u_2, u_3, u_4\}$, $A = \{u_1, u_3\}$ then (F_1, A) is AFS set defined as,

$$\delta_{F_1(u_1)} = \{(a_1, 0.2), (a_2, 0.9), (a_3, 0.5), (a_4, 0.9)\}$$

$$\delta_{F_1(u_3)} = \{(a_1, 0.4), (a_2, 0.8), (a_3, 0.6), (a_4, 0.8)\}$$

Hence (F_1, A) is AFS Γ-interior ideal and Γ-ideal over S.

Theorem.4.13. Let S be a AFS $\Gamma-$ semigroup. If S is regular then

$(F_1, A) = (F_1, A) \tilde{\circ} (S, E) \tilde{\circ} (F_1, A)$.

Proof. Since S is soft $\Gamma -$regular , $p \in S$ there exists $x \in S$ such that $p = p\alpha x\beta p$ for all $\alpha, \beta \in \Gamma$, $e \in A$.

Consider

$$(F_1(e) \tilde{\circ} S(e) \tilde{\circ} F_1(e))(p) = \inf_{x = y\Gamma z} \max\{(F_1(e) \tilde{\circ} S(e))(y), F_1(e)(z)\}$$

$$\le \max\{(F_1(e) \tilde{\circ} S(e))(p\alpha x), F_1(e)(p)\}$$

$$= \max\left\{\inf_{p\alpha x = a\Gamma b} \max\{F_1(e)(a) \tilde{\circ} S(e)(b)\},\ F_1(e)(p)\right\}$$

$$\le \max\{\max\{F_1(e)(p) \tilde{\circ} S(e)(x)\}, F_1(e)(p)\}$$

$$= \max\{\max\{F_1(e)(p) \tilde{\circ} 0\}, F_1(e)(p)\}$$

$$= \max\{F_1(e)(p),\ F_1(e)(p)\}$$

$$= F_1(e)(p)$$

Therefore $(F_{1,} A)\tilde{\circ}(S,E)\tilde{\circ}(F_{1,} A)\subseteq(F_{1,} A)$ and by theorem (3.11).

Hence $(F_{1,} A)\tilde{\circ}(S,E)\tilde{\circ}(F_{1,} A)=(F_{1,} A)$.

Acknowledgement:

The research of the second author is partially supported by UGC-BSR grant: F.25-1/2014-15(BSR)/7-254/2009(BSR) dated 20-01-5015 in India.

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