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A NUMERICAL METHOD OF SHAPE AND TOPOLOGICAL OPTIMIZATION USING LEVEL SET METHOD

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Abstract:

Topology is a major area of mathematics concerned with spatial properties that are preserved under continuous deformations of objects. In this paper a numerical method of shape and topological optimization structures based on the level set method and on shape differentiation is proposed. The level set method based on the classical shape derivative is known to handle boundary propagation with topological changes and this method can easily remove holes but cannot create new holes in the middle of a shape since the level set function obeys a maximum principle. The topological gradient method is precisely designed for introducing new holes in the optimization process. The gradient method is to applied to the minimization of classical shape derivative. Numerical methods based on the shape derivative may fall into a local minimum. The main idea is to test the optimality of a domain to topology variations by removing a small hole with appropriate boundary conditions. In this paper the structural optimization using topological and shape optimization is proved.

Key Words: Shape and Topological Optimization, Classical Shape, Level set method, Topological Gradient & Topological Derivative.

1. Introduction:

This paper is a numerical method of shape optimization based on the level set method and on shape differentiation. The level set method makes possible topology changes during the optimization process, it does not solve the inherent problem of ill posed ness of shape optimization which manifests itself in the frequent existence of many local (non global) minima, usually having different topologies. The reasons is that the level set method can easily remove holes but can not create new holes in the middle of a shape since the level set function obeys a maximum principle. In practice, this effect can be checked by varying the initialization which yields different optimal shapes with different topologies. To the best of our knowledge, other works on the level set method in shape optimization were also subject to this difficulty. The topological gradient method amounts to decide whether or not it is favorable (for decreasing the objective function) to nucleate a small hole in a given shape. As a matter of fact, creating a hole changes the topology and is thus one way of escaping local minima (due to topological constraint). For most of our 2-d numerical examples of compliance minimization, the expected global minimum is attained from the trivial full domain initialization. Nevertheless there are some (relatively few) examples of local minima if we choose a different initialization. For 2-d mechanism design our coupled method is not fully independent of several parameters, including initialization, although it already produces excellent results with the trivial full domain initialization. The main contribution of this paper is algorithmic and numeric. Our basic algorithm is to iteratively use the shape gradient or the topological gradient in a gradient based descent algorithm. We provide several 2-d and 3-d numerical examples for compliance minimization and mechanism design. In a slightly different context of inverse problems a different coupling of the shape and topological gradients (using the level set method too) has been proposed (8). There, the topological gradient was incorporated as a source term in the transport Hamilton-Jacobi equation used in the shape derivative algorithm for moving the shape.

2. Setting of the Problem:

A shape is bounded open set Ω C R^d (d=2 or 3) with a boundary made of two disjoint parts

$$\partial\Omega = \Gamma_N U \Gamma_D \qquad (2.1)$$

With Dirichlet boundary conditions on Γ_D, and Neumann boundary conditions on Γ_N. All admissible shapes Ω are required to a subset of a working domain *D*. The shape Ω is occupied by a linear isotropic elastic material with Hooke's law *A* defined, for any symmetric matrix ξ, by

$$A\,\xi = 2u\xi + \lambda\,(\mathrm{Tr}\xi)\,\mathrm{Id},$$

Where μ and λ are the Lame moduli of the material. The displacement field, u in Ω is the solution of the linearized elasticity system

$$\begin{aligned} -\mathrm{div}\,(A\,e(u)) &= f \text{ in } \Omega \\ u &= 0 \text{ on } \Gamma_D \end{aligned} \qquad (2.2)$$

$$(Ae(u)\, n = g \text{ on } \acute{\Gamma}_N$$

Where $f \in L2\ (D)^d$ and $g \in H^1\ (D)^d$ are the volume forces and the surface loads respectively. Assuming that $\acute{\Gamma}_D \neq \acute{\emptyset}$ (otherwise we should impose an equilibrium condition on f and g), (2.2) admits a unique solution in $u \in H^1\ (\Omega)^{d.}$

The objective function is denoted by J (Ω). Which is very common in rigidity maximization. A second choice is a least square error between and a target displacement.

$$J1(\Omega) = \int\Omega\ f.u\ dx + \int_{\Gamma}Ng.u\ ds = \int\Omega Ae(u).e(u)\ dx \qquad (2.3)$$

$$J1(\Omega) = (\int_{\Omega} k(x)\ u - u_0\backslash\alpha dx)^{1/\alpha} \qquad (2.4)$$

Which is a useful criterion for the design of compliant mechanisms. We assume $\alpha \geq 2$, a non negative given weighting factor. In both formulas (2.3) and (2.4) u = u(Ω) is the solution of (2.2). We define a set of admissible shapes that must be open sets contained in the working domain *D* and of fixed volume *V*

$$U_{ad}=\{\Omega C\ D \text{ such that } \Omega = V\}$$

A model problem of shape optimization is Inf J(Ω)

In practice we rather work with an unconstrained problem. Introducing a La grange multiplier *l*, we consider the Lagrangian minimization.

$$\text{Inf } L\ (\Omega) = J(\Omega) + L(\Omega)$$

3. Shape Derivative:

In order to apply a gradient method to the minimization of the classical notion of shape derivative, to Hadamard. Starting from a smooth reference open set Ω, consider domains of the type.

$$\Omega\theta = (Id + \theta)(\ \Omega) \qquad (3.1)$$

with Id the identity mapping from R^d into R^d and θ a vector field in (R^d, R^d). It is well known that, for sufficiently small θ, (ID+ θ) is a diffeomorphism in R^d. In other words, no change of topology is possible with this method of shape variation. The shape derivative of J(Ω) at Ω is defined as the Frechet derivative in $(R^d, R^{d)}$ at 0 of the application θ J((Id + θ) (Ω))⟶

$$J((Id + \theta)\ (\Omega)) = J(\Omega) + J'\ (\Omega)\ (\theta) + o(\theta)$$

4. Topological Derivative:

Numerical methods based on the shape derivative may therefore fall into a local minimum corresponding to the initial topology. The main idea is to test the optimality of a domain to topology variations by removing a small hole with appropriate boundary conditions.

5. Level Set Method for Shape Optimization:

Consider D C R^d a bounded domain in which all admissible shapes Ω are included, i.e. Ω C D. In numerical practice, the domain *D* will be uniformly meshed once and for all. We parameterize the boundary of Ω by means of a level set function, following the idea of Osher and sethian. We define this level set function Ψ in *D* such that

$$\begin{aligned} &\Psi\ (x) = 0, \quad X\ \varepsilon\ \partial\Omega \cap D \\ &\Psi\ (x) < 0, \quad X\ \varepsilon\ \Omega \qquad (5.1) \\ &\Psi\ (x) > 0, \quad X\ \varepsilon\ (D/\Omega) \end{aligned}$$

The normal to the shape Ω is recovered as $\Delta\Psi/|\Delta\Psi|$ and the mean curvature *H* is given by the divergence of the normal div $(\Delta\Psi/|\Delta\Psi|)$ these quantities are computer throughout the whole domain *D.* During the optimization process, the shape Ω(t) is going to evolve according to a fictitious time parameter t€R which corresponds to descent stepping. The evolution of the level set function is governed by the following Hamilton-Jacobi transport equation.

$$\delta\Psi / \delta t + V|\Delta\psi = 0 \quad \text{in } D \qquad (5.2)$$

Where *V* (t.x) is the normal velocity of the shape's boundary. Equation(5.2)is simply obtained by differentiating the definition of a level ser of ψ, ψ(t,x(t)) = Cst, and replacing the velocity *x(t)* by *Vn*.

The choice of the normal velocity *V* is based on the shape derivative

$$\pounds'(\Omega)(\theta) = \int v\theta.n\ ds \qquad (5.3)$$

Where the integrand v(u,p,n,H) depends on the state *u*, adjoint state *p*, normal *n* and mean curvature *H*. The simplest choice is to take the steepest descent. This yields a normal velocity for the shape's boundary *V = -v* (*v* is given everywhere in *D* and not only on the boundary ∂Ω).

6. Optimization Algorithm:

For the minimization problem we propose an iterative coupling of the level set method and of the topological gradient method. Both methods are gradient type algorithms, so our coupled method can be cast into the framework of alternate directions descent algorithms. The level set method relies on the shape derivative *L'* (Ω) of Section 3,while the topological gradient method is based on the topological derivative *DTL (x)*. These two types of derivative define independent descent directions that we simply alternate as follows. In a first step, the level set function ψ is advected according to the velocity. -v where v is the integrand in the shape derivative £'(Ω) see(5.3). In a second step, holes are introduced into the current domain Ω where the topological derivative DT£(x) is minimum and negative. More precisely, at those points we change the negative sign of the level set function ψ into a positive sign, according to the parameterization. In practice, it is better to perform more level set steps than topological gradient steps. Therefore, the main parameter of our coupled algorithm is an integer which is the number of gradient steps between two successive applications of the topological gradient.

Our Proposed Algorithm is an Iterative Method, Structured as Follows:

- ✓ Initialization of the level set function ψo corresponding to an initial guess Ωo (usually the full working domain *D*).
- ✓ Iteration until convergence, for k>0
  - **Elasticity Analysis**. Computation of the state u ,and adjoint p,state through two problems of linear elasticity posed in Ω. This yields the values of the shape derivative and of the topological gradient.
  - **Shape Gradient**. If mod (k,n) < n the current shape Ω characterized by the level set function ψ is deformed into a new shape Ω characterized ψ by which is the solution of the transport Hamilton Jacobi equation after a time interval with the initial condition and a velocity –v computed in terms of u and p ,. The time of integration Δ is chosen such that £(Ω) ≤ £(Ω).
  - **Topological Gradient.** If mod(k,n) = 0, we perform a nucleation step. We obtain a new shape Ω by inserting new holes into the current shape Ω. Namely, the sign of the level set function ψ is changed from negative to positive values in the regions of where the topological derivative depending on and has minimum negative values. If the objective function has in-creased.

The topological gradient step is performed only if the topological gradient is negative. If an infinitesimal small hole is inserted whereD£(x) < 0 , the objective function must decrease. However, in numerical practice, a hole can not be smaller than a single mesh cell, which is not so infinitesimally small. Even more, if the topological gradient is negative in several touching cells, it amounts to remove from the current shape a large zone which is not small at all.

7. Numerical Result:

We begin with single loads, minimal compliance problems, i.e. we minimize the Lagrangian

$$\text{Inf } \pounds(\Omega) = J(\Omega) + L(\Omega)$$

For a fixed positive Lagrange multiplier *l* > 0. The bridge problem is a 2 x 1.2 rectangle discredited with 3840 elements. The two lower corners have zero vertical displacement ants a unit vertical load is applied at the middle of its bottom. The Lagrange multiplier is. The initialization is the full domain. The coupling parameter is n = 5. The final result as well as the intermediate results where new holes are nucleated by the topological gradient are displayed on 7.1. This bridge problem is an example where local minima still exist despite the use of the topological gradient. Indeed, we run the same numerical example with a different initialization, namely the lower half of the domain. The resulting optimal shape, displayed on figure 7.2.

8. Conclusion:

We have proposed a coupled method of shape and topology differentiation in the level set framework. It is an iterative algorithm where repeatedly the shape boundary evolves smoothly and new small holes are nucleated. In numerical practice, this method is more insensitive to the initialization and is thus a great improvement over the level set method .By removing a hole in a shape is not the only possibility for changing the topology. It is somehow the opposite process of hole perforation, since it adds some material to the shape, Numerically this could be an interesting process that may avoid, for example, the different optimal shapes obtained for the 2-d bridge problem. Finally, we remark that, for compliance minimization problems, the homogenization method is still the most reliable method since it is only one which is fully independent of the initialization and free of any important parameters.

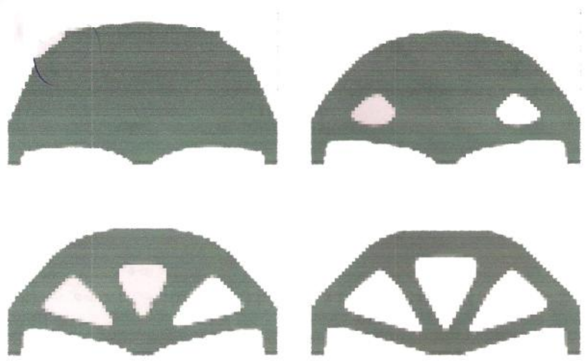

Figure 7.1: Optimal bridge in 2-d: iteration 6, 11 and 100

Figure 7.2: Optimal bridge in 2 d: half domain initialization and optimal shape after 100 iterations.

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