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LEVEL OPERATORS ON INTUITIONISTIC L-FUZZY SETS

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Abstract:

In this paper, we propose the Level Operators on Intuitionistic L-Fuzzy Sets and establish some of their properties.

Key Words: Fuzzy Set, Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Sets Of Second Type & Intuitionistic L-Fuzzy Set

1. Introduction:

Fuzzy sets were introduced by Lotfi. A. Zadeh in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets *[IFS]* and their extensions like Intuitionistic Fuzzy sets of second type *[IFSST]*, Intuitionistic L-Fuzzy sets *[ILFS]* and Temporal Intuitionistic Fuzzy sets *[TIFS]*. In section 2, we give some basic definitions and in section 3, we introduce the Level operators on *ILFS* in the universal set E and establish some of their properties. The paper is concluded in section 4.

2. Preliminaries:

In this section, we give some basic definitions.

Definition 2.1: A Fuzzy set *[FS]*A in a universal set E is defined by

$$A = \{\langle x, \mu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ is the membership function representing the membership degree of element x in the *FS* A such that $0 \leq \mu_A(x) \leq 1$.

Definition 2.2: An Intuitionistic Fuzzy set*[IFS]*A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x$in E respectively, satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

The value $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the degree of uncertainty of the element $x \in E$ to the *IFS* A.

Definition 2.3: An Intuitionistic Fuzzy sets of second type*[IFSST]*A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in E$ respectively, satisfying $0 \leq \mu_A(x)^2 + \nu_A(x)^2 \leq 1$.

The value $\pi_A(x) = \sqrt{1 - \mu_A(x)^2 + \nu_A(x)^2}$is the degree of uncertainty of the element $x \in E$ to the *IFSST* A.

Definition 2.4: An Intuitionistic L-Fuzzy set *[ILFS]* A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

where $\mu_A: E \to L$ and $\nu_A: E \to L$ denote the degree of membership and the degree of non-membership of the element $x \in E$ respectively, satisfying $\mu_A(x) \leq N(\nu_A(x))$, $N: L \to L$ is an unary involute order reversing operation and E be fixed.

The value $\pi_A(x) = N(\sup((\mu_A(x), \nu_A(x))))$ is the degree of uncertainty of the element $x \in E$ to the *ILFS* A.

3. Level Operators on Intuitionistic L-Fuzzy Sets:

In this section, we define the Level operators on *ILFS* in the universal set E and establish some of their properties.

The following two operators $!(A)$ and $?(A)$

$$!(A) = \left\{\langle x, \sup\left(\frac{1}{2}, \mu_A(x)\right), \inf\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\},$$
$$?(A) = \left\{\langle x, \inf\left(\frac{1}{2}, \mu_A(x)\right), \sup\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\},$$

are called the Level Operators on the *ILFS* A.

Example 3.1:

Let $E = \{a, b, c\}$and Let the *ILFS* A is,

$$A = \{\langle a, 0.4, 0.3\rangle, \langle b, 0.7, 0.2\rangle, \langle c, 0.2, 0.6\rangle\}$$
then $$!A = \{\langle a, 0.5, 0.3\rangle, \langle b, 0.7, 0.2\rangle, \langle c, 0.5, 0.5\rangle\}$$
and $$?A = \{\langle a, 0.4, 0.5\rangle, \langle b, 0.5, 0.5\rangle, \langle c, 0.2, 0.6\rangle\}$$

are the Level Operators on A.

Theorem 3.1: For any *ILFS* A, we have

(a) $!(A) = \overline{?\,\overline{(A)}}$,

(b) $!(A) = A \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\}$,

(c) $?(A) = A \cap \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\}$,

(d) $!(?(A)) = ?(!(A)) = \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\}$.

Proof:

(a) $\overline{?\,\overline{(A)}} = \overline{?\{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\}}$

$\overline{?\,\overline{(A)}} = \overline{\left\{\langle x, \inf\left(\frac{1}{2}, \nu_A(x)\right), \sup\left(\frac{1}{2}, \mu_A(x)\right)\rangle / x \in E\right\}}$

$\overline{?\,\overline{(A)}} = \left\{\langle x, \sup\left(\frac{1}{2}, \mu_A(x)\right), \inf\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\}$

$\overline{?\,\overline{(A)}} = !(A)$.

(b) $A \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = \{\langle x, \mu_A(x), \nu_A(x)\rangle x \in E\} \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle x \in E\right\}$

$A \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = \left\{\langle x, sup\left(\mu_A(x), \frac{1}{2}\right), inf\left(\nu_A(x), \frac{1}{2}\right)\rangle / x \in E\right\}$

$A \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = \left\{\langle x, sup\left(\frac{1}{2}, \mu_A(x)\right), inf\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\}$

$A \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = !(A)$.

(c) $A \cap \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\} \cap \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\}$

$A \cap \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = \left\{\langle x, inf\left(\mu_A(x), \frac{1}{2}\right), sup\left(\nu_A(x), \frac{1}{2}\right)\rangle / x \in E\right\}$

$A \cap \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = \left\{\langle x, inf\left(\frac{1}{2}, \mu_A(x)\right), sup\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\}$

$A \cup \left\{\langle x, \frac{1}{2}, \frac{1}{2}\rangle / x \in E\right\} = ?(A)$.

(d) $!(?(A)) = !\left(\left\{\langle x, inf\left(\frac{1}{2}, \mu_A(x)\right), sup\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\}\right)$

$!(?(A)) = \left\{\langle x, sup\left(\frac{1}{2}, inf\left(\frac{1}{2}, \mu_A(x)\right)\right), inf\left(\frac{1}{2}, sup\left(\frac{1}{2}, \nu_A(x)\right)\right)\rangle / x \in E\right\}$

$$!\left(?\,(A)\right) = \left\{\langle x, \tfrac{1}{2}, \tfrac{1}{2}\rangle \,/x \in E\right\} \qquad (1)$$

$$?\left(!\,(A)\right) = ?\left(\left\{\langle\, x, sup\left(\tfrac{1}{2}, \mu_A(x)\right), inf\left(\tfrac{1}{2}, \nu_A(x)\right)\rangle \,/x \in E\right\}\right)$$

$$?\left(!\,(A)\right) = \left\{\langle x, inf\left(\tfrac{1}{2}, sup\left(\tfrac{1}{2}, \mu_A(x)\right)\right), sup\left(\tfrac{1}{2}, inf\left(\tfrac{1}{2}, \nu_A(x)\right)\right)\rangle \,/x \in E\right\}$$

$$?\left(!\,(A)\right) = \left\{\langle x, \tfrac{1}{2}, \tfrac{1}{2}\rangle \,/x \in E\right\} \qquad (2)$$

From (1) and (2), we have $!\left(?\,(A)\right) = ?\left(!\,(A)\right) = \left\{\langle x, \tfrac{1}{2}, \tfrac{1}{2}\rangle \,/x \in E\right\}$.

Theorem 3.2: For every two $ILFS$ A and B, we have

(a) $!\,(A \cap B) = !\,(A) \cap !\,(B)$,
(b) $!\,(A \cup B) = !\,(A) \cup !\,(B)$,
(c) $?\,(A \cup B) = ?\,(A) \cup ?\,(B)$,
(d) $?\,(A \cap B) = ?\,(A) \cap ?\,(B)$.

Proof:

(a) $!\,(A \cap B) = !\left(\left\{\langle x, inf\left(\mu_A(x), \mu_B(x)\right), sup\left(\nu_A(x), \nu_B(x)\right)\rangle / \,x \in E\right\}\right)$

$$!\,(A \cap B) = \left\{\langle x, sup\left(\tfrac{1}{2}, inf\left(\mu_A(x), \mu_B(x)\right)\right), inf\left(\tfrac{1}{2}, sup\left(\nu_A(x), \nu_B(x)\right)\right)\rangle \,/x \in E\right\}$$

$$!\,(A \cap B) = \left\{\langle x, inf\left(sup\left(\tfrac{1}{2}, \mu_A(x)\right), sup\left(\tfrac{1}{2}, \mu_B(x)\right)\right), sup\left(inf\left(\tfrac{1}{2}, \nu_A(x)\right), inf\left(\tfrac{1}{2}, \nu_B(x)\right)\right)\rangle / \,x \in E\right\}$$

$$!\,(A \cap B) = \left\{\langle x, sup\left(\tfrac{1}{2}, \mu_A(x)\right), inf\left(\tfrac{1}{2}, \nu_A(x)\right)\rangle \,/x \in E\right\} \cap \left\{\langle x, sup\left(\tfrac{1}{2}, \mu_B(x)\right), inf\left(\tfrac{1}{2}, \nu_B(x)\right)\rangle \,/x \in E\right\}$$

$$!\,(A \cap B) = !\,(A) \cap !\,(B).$$

(b) $!\,(A \cup B) = !\left(\left\{\langle x, sup\left(\mu_A(x), \mu_B(x)\right), inf\left(\nu_A(x), \nu_B(x)\right)\rangle / \,x \in E\right\}\right)$

$$!\,(A \cup B) = \left\{\langle x, sup\left(\tfrac{1}{2}, sup\left(\mu_A(x), \mu_B(x)\right)\right), inf\left(\tfrac{1}{2}, inf\left(\nu_A(x), \nu_B(x)\right)\right)\rangle \,/x \in E\right\}$$

$$!\,(A \cup B) = \left\{\langle x, sup\left(sup\left(\tfrac{1}{2}, \mu_A(x)\right), sup\left(\tfrac{1}{2}, \mu_B(x)\right)\right), inf\left(inf\left(\tfrac{1}{2}, \nu_A(x)\right), inf\left(\tfrac{1}{2}, \nu_B(x)\right)\right)\rangle / \,x \in E\right\}$$

$$!\,(A \cap B) = \left\{\langle x, sup\left(\tfrac{1}{2}, \mu_A(x)\right), inf\left(\tfrac{1}{2}, \nu_A(x)\right)\rangle \,/x \in E\right\} \cup \left\{\langle x, sup\left(\tfrac{1}{2}, \mu_B(x)\right), inf\left(\tfrac{1}{2}, \nu_B(x)\right)\rangle \,/x \in E\right\}$$

$$!\,(A \cup B) = !\,(A) \cup !\,(B).$$

(c) $?\,(A \cap B) = ?\left(\left\{\langle x, inf\left(\mu_A(x), \mu_B(x)\right), sup\left(\nu_A(x), \nu_B(x)\right)\rangle /x \in E\right\}\right)$

$$?\,(A \cap B) = \left\{\langle x, inf\left(\tfrac{1}{2}, inf\left(\mu_A(x), \mu_B(x)\right)\right), sup\left(\tfrac{1}{2}, sup\left(\nu_A(x), \nu_B(x)\right)\right)\rangle \,/x \in E\right\}$$

$$?\,(A \cap B) = \left\{\langle x, inf\left(inf\left(\tfrac{1}{2}, \mu_A(x)\right), inf\left(\tfrac{1}{2}, \mu_B(x)\right)\right), sup\left(sup\left(\tfrac{1}{2}, \nu_A(x)\right), sup\left(\tfrac{1}{2}, \nu_B(x)\right)\right)\rangle / \,x \in E\right\}$$

$$?(A\cap B)=\left\{\langle x, inf\left(\frac{1}{2},\mu_A(x)\right), sup\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}\cap \left\{\langle x, inf\left(\frac{1}{2},\mu_B(x)\right), sup\left(\frac{1}{2},\nu_B(x)\right)\rangle /x\in E\right\}$$

$$?(A\cap B)=?(A)\cap ?(B).$$

(d) $$?(A\cup B)=?\left(\left\{\langle x, sup(\mu_A(x),\mu_B(x)), inf(\nu_A(x),\nu_B(x))\rangle /x\in E\right\}\right)$$

$$?(A\cup B)=\left\{\langle x, inf\left(\frac{1}{2}, sup(\mu_A(x),\mu_B(x))\right), sup\left(\frac{1}{2}, inf(\nu_A(x),\nu_B(x))\right)\rangle /x\in E\right\}$$

$$?(A\cup B)=\left\{\langle x, sup\left(inf\left(\frac{1}{2},\mu_A(x)\right), inf\left(\frac{1}{2},\mu_B(x)\right)\right), inf\left(sup\left(\frac{1}{2},\nu_A(x)\right), sup\left(\frac{1}{2},\nu_B(x)\right)\right)\rangle /x\in E\right\}$$

$$?(A\cup B)=\left\{\langle x, inf\left(\frac{1}{2},\mu_A(x)\right), sup\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}\cup \left\{\langle x, inf\left(\frac{1}{2},\mu_B(x)\right), sup\left(\frac{1}{2},\nu_B(x)\right)\rangle /x\in E\right\}$$

$$?(A\cup B)=?(A)\cup ?(B).$$

Theorem 3.3: For any $ILFS\ A$, we have

(a) $\square !A = !\square A$,

(b) $\square ?A = ?\square A$,

(c) $\Diamond !A = !\Diamond A$,

(d) $\Diamond ?A = ?\Diamond A$.

Proof:

(a) $$\square !A=\square\left(\left\{\langle x, sup\left(\frac{1}{2},\mu_A(x)\right), inf\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}\right)$$

$$\square !A=\left\{\langle x, sup\left(\frac{1}{2},\mu_A(x)\right), N\left(sup\left(\frac{1}{2},\mu_A(x)\right)\right)\rangle /x\in E\right\}$$

$$\square !A=\left\{\langle x, sup\left(\frac{1}{2},\mu_A(x)\right), inf\left(\frac{1}{2}, N(\mu_A(x))\right)\rangle /x\in E\right\}$$

$$\square !A = !\square A.$$

(b) $$\square ?A=\square\left(\left\{\langle x, inf\left(\frac{1}{2},\mu_A(x)\right), sup\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}\right)$$

$$\square ?A=\left\{\langle x, inf\left(\frac{1}{2},\mu_A(x)\right), N\left(inf\left(\frac{1}{2},\mu_A(x)\right)\right)\rangle /x\in E\right\}$$

$$\square ?A=\left\{\langle x, inf\left(\frac{1}{2},\mu_A(x)\right), sup\left(\frac{1}{2}, N(\mu_A(x))\right)\rangle /x\in E\right\}$$

$$\square ?A = ?\square A.$$

(c) $$\Diamond !A=\Diamond\left(\left\{\langle x, sup\left(\frac{1}{2},\mu_A(x)\right), inf\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}\right)$$

$$\Diamond !A=\left\{\langle x, N\left(inf\left(\frac{1}{2},\nu_A(x)\right)\right), inf\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}$$

$$\Diamond !A=\left\{\langle x, sup\left(\frac{1}{2}, N(\nu_A(x))\right), inf\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}$$

$$\Diamond !A = !\Diamond A.$$

(d) $$\Diamond ?A=\Diamond\left(\left\{\langle x, inf\left(\frac{1}{2},\mu_A(x)\right), sup\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}\right)$$

$$\Diamond ?A=\left\{\langle x, N\left(sup\left(\frac{1}{2},\nu_A(x)\right)\right), sup\left(\frac{1}{2},\nu_A(x)\right)\rangle /x\in E\right\}$$

$◊?A = \left\{\langle x, inf\left(\frac{1}{2}, N\left(\nu_A(x)\right)\right), sup\left(\frac{1}{2}, \nu_A(x)\right)\rangle / x \in E\right\}$

$◊?A = ?◊A.$

4. Conclusion:

In this paper, we have introduced the Level operators on Intuitionistic L-Fuzzy sets *[ILFS]* and established some of their properties. In future we will study some more operators on *ILFS* and their applications.

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