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ON SOME DISTANCE MEASURES IN INTUITIONISTIC FUZZY SETS

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Abstract:

In this paper, we gave a lucid and comprehensive on some selected models of IFSs in real life situations such as in career determination and pattern recognition using normalized Euclidean distance measure.

Key Words: Intuitionistic Fuzzy Sets, Decision Making, Career Determination, Pattern Recognition & Distance between Intuitionistic Fuzzy Sets

1. Introduction:

Fuzzy sets (FS) introduced by [1] has showed meaningful applications in many fields of study. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree.

Therefore, a generalization of fuzzy sets was proposed by [2, 3] as intuitionistic fuzzy sets (IFS) which incorporate the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degrees respectively). The interesting and useful in many application areas. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and hesitation margin [4, 6]. Szmidt and Kacprzk [8] showed that intuitionistic fuzzy sets are pretty useful in situations when description of a problem by a linguistic variable given in terms of a membership function only seems to rough. Due to the flexibility of IFS in handling uncertainly, they are tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge.

Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world, such as in machine learning, medical diagnosis, electoral system, career determination and pattern recognition and so on.

In this paper, we present some applications of intuitionistic fuzzy sets in career determination and pattern recognition using normalized Euclidean distance measure proposed in [5, 7, 9, 10, 11].

2. Concept of Intuitionistic Fuzzy Sets:

Definition 1: (Zadeh, 1965) Let X be a non-empty. A fuzzy set 'A' drawn from X is defined as $A = \{\langle x,\ \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x): X \to [0,1]$ is the membership function of the fuzzy set 'A'. Fuzzy set is a collection of objects with graded membership (i.e.) having degrees of membership.

Definition 2: (Atanassov 1999) Let X be a non-empty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x,\ \mu_A(x),\ v_A(x)\rangle : x \in X\}$, Where the functions $\mu_A(x),\ v_A(x): X \to [0, 1]$ define respectively, the degree of membership and non-membership of the element $x \in X$ to the set A, which is a subset of X and for every element $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1 \quad \forall\, x \in X$. Furthermore, We have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index (or) hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ (i.e.) $\pi_A(x): X \to [0,1]$ and $0 \le \pi_A(x) \le 1 \quad \forall\, x \in X$. $\pi_A(x)$ Expresses the lack of knowledge of whether x belongs to IFS A or not.

3. Some Distance Measures in Intuitionistic Fuzzy Sets:

Definition 3: Let 'X' be non-empty set. Such that IFS $A, B, C \in X$. Then the distance measure 'd' between IFS A and B is a mapping $d: X \times X \to [0,1]$;

if $d(A,B)$ satisfies the following axioms

A_1. $0 \le d(A,B) \le 1$

A_2. $d(A,B) = 0 \Leftrightarrow A = B$

A_3. $d(A,B) = d(B,A)$

A_4. $d(A,C) + d(B,C) \ge d(A,B)$;

A_5. if $A \subseteq B \subseteq C$

Then $d(A,C) \ge d(A,B)$ and $d(A,C) \ge d(B,C)$

We make use of the four distance measures proposed. Let, $A = \{\langle x, \mu_A(x_i), \nu_A(x_i), \pi_A(x_i)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i), \pi_B(x_i)\rangle : x \in X\}$ be two IFS'S in $X = \{x_1, x_2, \ldots\ldots x_n\}$, $i = 1, 2, \ldots.n$ based on the geometric interpretation of IFS Szmidt and Kacprzyk [5, 7, 9] proposed.

1. Hamming Distance:

$$d_H(A,B) = \frac{1}{2}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right)$$

2. The Euclidean Distance:

$$d_E(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\pi_A(x_i) - \pi_B(x_i)\right)^2\right]}$$

3. The Normalized Hamming Distance:

$$d_{n-H}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right)$$

4. The Normalized Euclidean Distance:

$$d_{n-E}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\pi_A(x_i) - \pi_B(x_i)\right)^2\right]}$$

Example: Let, $A = \{\langle 0.6, 0.2, 0.2\rangle\langle 0.5, 0.3, 0.2\rangle\}$ and $B = \{\langle 0.5, 0.4, 0.1\rangle\langle 0.4, 0.1, 0.5\rangle\}$ be IFS's in X. $X = \{x_1, x_2\}$ we use the above distance measures to calculate the distance between A and B.

- ✓ Hamming distance of A and B (i.e.) $d_H(A,B) = 0.500$,
- ✓ Euclidean distance of A and B (i.e.) $d_E(A,B) = 0.250$
- ✓ Normalized hamming distance of A and B (i.e.) $d_{n-H}(A,B) = 0.316$
- ✓ Normalized Euclidean distance of A and B (i.e.) $d_{n-E}(A,B) = 0.224$

From these results, we saw that the normalized Euclidean distance gives the best distance measure between A and B. For this reason, we shall make use of normalized Euclidean distance in the applications for its high rate of confidence in terms of accuracy.

4. Experimental Results:

The notion of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. This idea of IFS seems to be resourceful in modeling many real life situations like negotiation processes, psychological investigations, reasoning, medical diagnosis among others. We show a novel application of intuitionistic fuzzy set career choice. An example of career determination will be presented, assuming there is a database. We will describe the state of students knowing the results of their performance. The problem description uses the concept of IFS that makes it possible to render two important facts. First, Values of each subject performance changes for each careers. Second, values of each student's for subject performance. We use the the normalized Euclidean distance measure method given in [5, 7, 10] to measure the distance between each student and each career.

5. Model of Intuitionistic Fuzzy Sets in Career Determination:

The essence of providing adequate information to students for proper career choice cannot be overemphasized. This is paramount because the numerous problems of lack of proper career guide faced by students are of great consequence on their career choice and efficiency. Therefore, it is expedient that students be given sufficient information on career determination (or) choice to enhance adequate planning, preparation and proficiency. Among the career determining factors such as academic performance, personality make-up etc.

The first mentioned seems to be overriding we use intuitionistic fuzzy sets as tool since it incorporates the membership degree (i.e., the marks of questions answered by the student), the non-membership degree (i.e., the marks allocated to the questions the student failed) and the hesitation degree (which is the mark allocated to the questions the student do not attempt) Let, $S = \{S_1, S_2, S_3, S_4\}$ be the set of students, C= $\{medicine,\ pharmacy, surgery, anatomy\}$ be the set of Careers and $S_\mu = \{English\ Language,\ Mathematics,\ Biology,\ Physics,\ chemistry\}$ be the set of subjects related to the careers. We assume the above students sit for examinations on the above mentioned subjects to determine their career placements and choices. The table below shows careers and related subjects requirements in terms of intuitionistic fuzzy values.

Table 1: Careers Vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
S_1	(0.9, 0.0, 0.1)	(0.4, 0.2, 0.4)	(0.6, 0.2, 0.2)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)
S_2	(0.9, 0.0, 0.1)	(0.4, 0.2, 0.4)	(0.6, 0.2, 0.2)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)
S_3	(0.7, 0.3, 0.0)	(0.6, 0.2, 0.2)	(0.9, 0.0, 0.1)	(0.5, 0.4, 0.1)	(0.5, 0.3, 0.2)
S_4	(0.5, 0.3, 0.2)	(0.5, 0.2, 0.3)	(0.9, 0.0, 0.1)	(0.7, 0.2, 0.1)	(0.7, 0.1, 0.2)

Each performance is described by 3 numbers. (i.e.,) membership μ, Non-membership v, and hesitation margin π. After the various examinations, the students obtained the following marks as shown in the table below

Table 2: Students Vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
Medicine	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.2, 0.2)	(0.5, 0.3, 0.2)	(0.5, 0.5, 0.0)
Pharmacy	(0.5, 0.3, 0.2)	(0.6, 0.2, 0.2)	(0.5, 0.3, 0.2)	(0.4, 0.5, 0.1)	(0.7, 0.2, 0.1)
Surgery	(0.7, 0.1, 0.2)	(0.6, 0.3, 0.1)	(0.7, 0.1, 0.2)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)
Anatomy	(0.6, 0.4, 0.0)	(0.8, 0.1, 0.1)	(0.6, 0.0, 0.4)	(0.6, 0.3, 0.1)	(0.5, 0.3, 0.2)

Table 3: Students Vs Careers

	Medicine	Pharmacy	Surgery	Anatomy
S_1	0.200	0.251	0.210	0.232
S_2	0.245	0.214	0.237	0.272
S_3	0.268	0.249	0.219	0.219
S_4	0.265	0.235	0.263	0.187

Using normalized Euclidean distance formula to calculate the distance between each student and each career with reference to the subjects, we get the table above.

From the above table, the shortest distance gives the proper career determination. S_1 is to read Medicine (doctor), S_2 is to read Pharmacy (Pharmacist), S_3 is to read either Surgery (surgeon) (or) Anatomy (Anatomist), S_4 is to read Anatomy (Anatomist).

6. Model of Intuitionistic Fuzzy Sets in Pattern Recognition:

In this process, a set of patterns is given (intuitionistic in nature), and another, unknown pattern called sample is given (also intuitionistic in nature). Both the set of the pattern and that of the sample are within the same feature space (or) attributes 'n'. The task is to find the distance between each of the patterns and the sample. The smallest (or) shortest distance between any of the patterns and the sample. Shows that, the sample belongs to that pattern. This is what pattern recognition is all about. Assume that, 'n' Patterns given by

$$A_j = \{\langle x_i,\ \mu_A(x_i), v_A(x_i),\ \pi_A(x_i)\rangle : x_i \in X\} \quad i = 1, 2, 3 \ldots \ldots . . n$$

and $A_j = \{A_1, A_2, \ldots \ldots \ldots A_m\}$ for $m \in N$

Suppose that, there is a sample to be recognized, that is

$$B = \{\langle x_i,\ \mu_B(x_i),\ v_B(x_i),\ \pi_B(x_i)\rangle : x_i \in X\} \quad i = 1, 2, 3 \ldots \ldots . . n$$

Example: Let, six patterns be represented by IFS's in $X = \{x_1, x_2,\ x_3, x_4\}$ ((i.e.,) feature space for $= 4$) as

$$A_1 = \{\langle 1.0, 0.0, 0.0\rangle, \langle 0.8, 0.1, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.5, 0.3, 0.2\rangle\},$$
$$A_2 = \{\langle 0.8, 0.1, 0.1\rangle, \langle 1.0, 0.0, 0.0\rangle, \langle 0.9, 0.1, 0.0\rangle, \langle 0.6, 0.1, 0.3\rangle\},$$
$$A_3 = \{\langle 0.6, 0.2, 0.2\rangle, \langle 0.8, 0.0, 0.2\rangle, \langle 1.0, 0.0, 0.0\rangle, \langle 0.8, 0.1, 0.1\rangle\},$$
$$A_4 = \{\langle 0.8, 0.2, 0.0\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.8, 0.2, 0.0\rangle, \langle 0.5, 0.3, 0.2\rangle\},$$
$$A_5 = \{\langle 0.6, 0.3, 0.1\rangle, \langle 0.9, 0.1, 0.0\rangle, \langle 1.0, 0.0, 0.0\rangle, \langle 0.8, 0.0, 0.2\rangle\},$$

and $A_6 = \{\langle 0.9, 0.1, 0.0\rangle, \langle 0.8, 0.1, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.5, 0.3, 0.2\rangle\},$

be the classification of building materials. Consider, another kind of unknown building material is $B = \{\langle 0.5, 0.3, 0.2\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.8, 0.1, 0.1\rangle, \langle 0.9, 0.1, 0.0\rangle\},\ A, B \in X$

Our task is to show which class of A_j (for $j = 1, 2, 3, 4, 5, 6$), the unknown pattern B belongs to using 4, We have the following results: $d_{n-E}(A_1, B) = 0.270$, $d_{n-E}(A_2, B) = 0.255$ $d_{n-E}(A_3, B) = 0.146$, $d_{n-E}(A_4, B) = 0.221$, $d_{n-E}(A_5, B) = 0.177$, $d_{n-E}(A_6, B) = 0.257$

From these results, we see that, the distance A_3 and B is the smallest and the distance between A_1 and B is the greatest. Since, A_3 approaches B, we say that the unknown pattern B belongs to A_3.

7. Conclusion:

In this paper, first we have discussed model of career determination using normalized Euclidean distance to calculate the distance of each student from each career in respect to the subject, solution is obtained by looking for the smallest distance between each student and career. Second we have discussed model of pattern recognition using normalized Euclidean distance to calculate the smallest (or) shortest distance between any of the patterns and the sample. The sample belongs to that pattern, obtained results. Further we are going to establish more results on decision making problems.

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