International Journal of Scientific Research and Modern Education (IJSRME)
ISSN (Online): 2455 – 5630
(www.rdmodernresearch.com) Volume I, Issue II, 2016

PRIME LABELING FOR SOME SUNLET RELATED GRAPHS

V. Ganesan* & Dr. K. Balamurugan**
* Assistant Professor of Mathematics, T.K Government Arts College, Vriddhachalam, Tamilnadu
** Associate Professor of Mathematics, Government Arts College, Thiruvannamalai, Tamilnadu

Abstract:

A graph G with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots . |V|$ *such that for each xy the labels assigned to x and y are relatively prime.A graph which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling for some sunlet related graphs. We also discuss prime labeling in the context of some graph operations namely fusion, duplication, switching and path union.*

Key Words: Prime Labeling, Fusion, Duplication, Switching & Path Union

1. Introduction:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and the edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to J. A. Bondy and U.S.R. Murthy [1]. In the present work S_n denotes sunlet graph with $2n$ vertices. We give brief summary of definitions which are useful for the present investigation. Enough literatures available in printed as well as electronics form on different types of graph labeling and more than 1000 research papers have been published so far in past four decades. A current survey of various graphs labeling problem can be found in [4] (J. Gallian, 2009)

Following are the common features of any graph Labeling problem.

- ✓ A set of numbers from which vertex labels are assigned.
- ✓ A rule that assigns value to each edge.
- ✓ A condition that these values must satisfy.

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout (1982 P 365-368) [7]. Many researches have studied prime graph for example in H.C. Fu(1994 P 181-186) [3] have proved that path P_n on n vertices is a prime graph.

T.O. Dertsky (1991 P 359-369) [2] have proved that the C_n on n vertices is a prime graph. S.M. Lee (1998 P 59 -67) [5] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram (2006, P205-209) [6]. In [8] S.K. Vaidhya and K. K. Kanmani) have proved that the prime labeling for some cycle related graphs.

Definition 1.1: If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition 1.2: Let $G = \big(V(G), E(G)\big)$ be a graph with n vertices. A bijection $f : V(G) \rightarrow \{1, 2, \ldots n\}$ is called a Prime labeling if for each edge $e = uv$, $gcd\big(f(u), f(v)\big) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.3: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 1.4: The sunlet graph S_n is the graph obtained from a cycle C_n attaching a pendant edge at each vertex of the n-cycle.

Example:

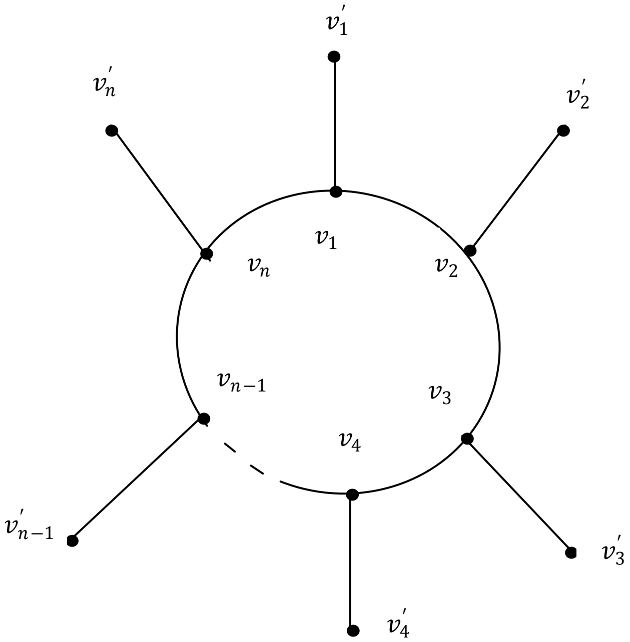


Figure1: A Sunlet graph S_n

Definition1.5: Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex $x$inG_1 such that every edge which was incident with either u (or) $v$in G now incident with x in G_1.

Definition 1.6: Duplication of a vertex v_k of a graph G produces a new graph $G_1$by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words, a vertex v_k' is said to be a duplication ofv_k if all the vertices which are adjacent to v_k are now adjacent to v_k'.

Definition 1.7: A vertex switching G_v of a graph $G$is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.8: Let $G_1, G_2, G_3, \ldots G_n$, $n \geq 2$ be$n$ copies of a fixed graph G. The graph obtained by adding an edge between G_i and$G_{i+1}$for $i = 1, 2, \ldots n-1$ is called the path union of G.

2. Main Results:

Proposition 2.1:

The sunlet graph S_n is a prime graph.

Proof:

Let $S_n$be a sunlet graph with $2n$ vertices

and $V(S_n) = \{v_1, v_2, \ldots\ldots v_n, v_1', v_2', \ldots. v_n'\}$

$E(S_n) = \{v_i v_i' / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_1 v_n\}$

Here$|V(S_n)| = 2n$ when $n$is even and odd.

Let $f(v_i) = 2i + 1$ for $0 \leq i \leq n-1$ and

$f(v_i') = 2i$ for $1 \leq i \leq n$

Since each edge $e = v_i v_{i+1} \in G$, $\gcd(f(v_i), f(v_{i+1})) = 1$

And $e = v_i v_i' \in G$, $\gcd\left(f(v_i), f(v_i')\right) = 1$

$\therefore f$ admits a prime labeling,

Thus S_n is a prime graph.

Example:

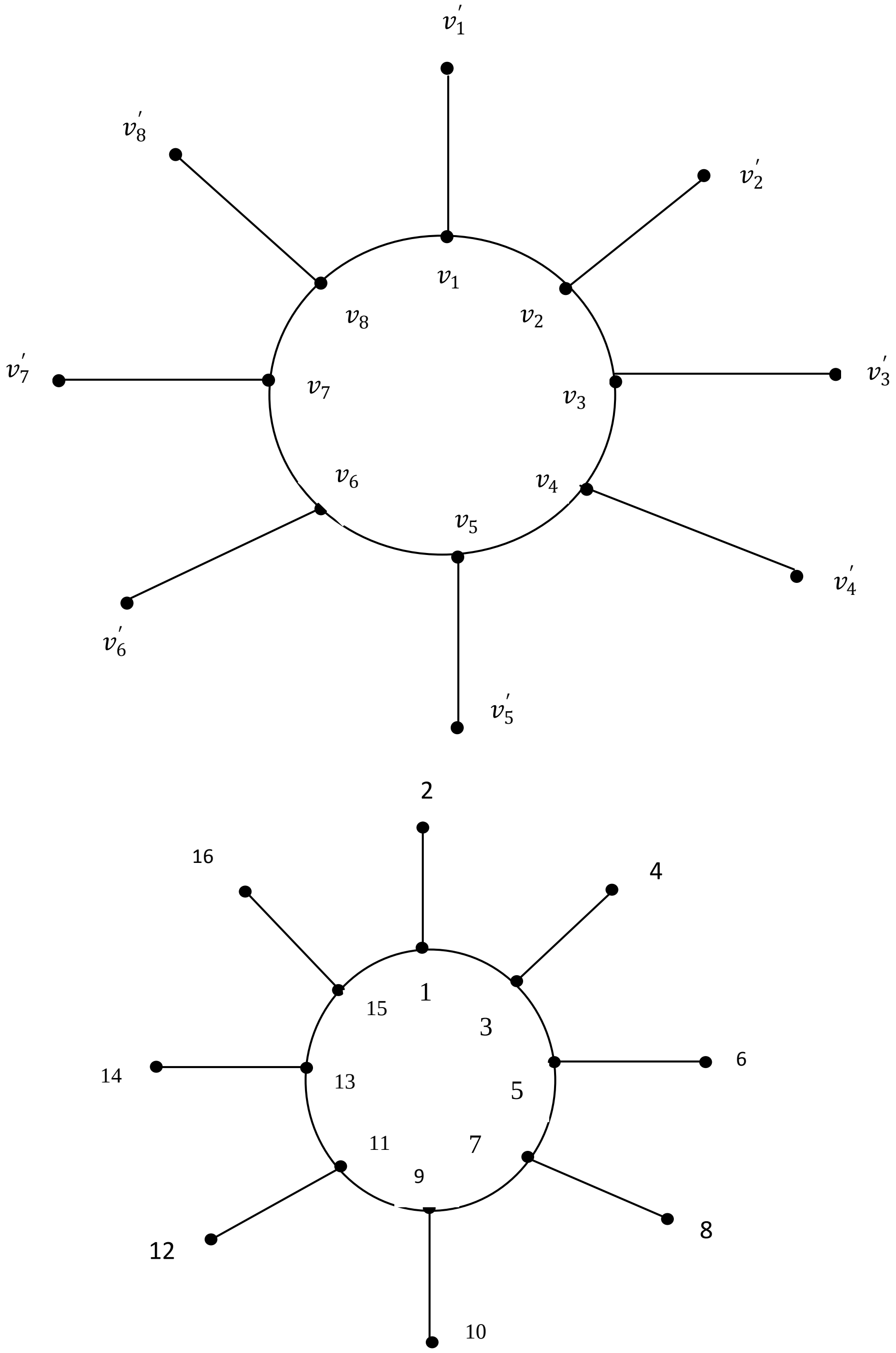


Figure 2: Prime labeling of sunlet graph S_8

Proposition 2.2:

The graph obtained by fusing any two consecutive vertices in a sunlet graph S_n is a prime graph.

Proof:

Let, $V(S_n) = \{v_{1,}v_2, \ldots\ldots v_n, v_1', v_2', \ldots. v_n'\}$

$E(S_n) = \{v_i v_i' / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_1 v_n\}$

Here, $|V(S_n)| = 2n$

Let G be the graph obtained after fusion of two vertices.

$\therefore \quad |V(G)| = 2n - 1$

Case (i):

When fusing any two consecutive vertices in v_i of S_n.

Define a labeling $f: V(G) \rightarrow \{1, 2, ..2n-1\}$ as follows.

Let $f(v_1) = 2$ where v_1 is a new vertex (after fused)

$f(v_i^{'}) = 1, 3$ where $v_i^{'}$ are pendant vertices incident with v_1

$f(v_i) = 2i + 1$ for $2 \leq i \leq n-1$

$f(v_i^{'}) = 2i$ for $2 \leq i \leq n-1$

Case (ii):

When fusing any two consecutive vertices one in v_i and other $v_i^{'}$ of S_n.

Define a labeling $f: V(G) \rightarrow \{1,2, ..2n-1\}$ as follows.

Let $f(v_1) = 1$ where v_1 is a new vertex

$f(v_i) = 2i + 1$ for $2 \leq i \leq n-1$

$f(v_i^{'}) = 2i$ for $1 \leq i \leq n-1$

Then for each f admits a prime labeling

Thus G is a prime graph.

Example:

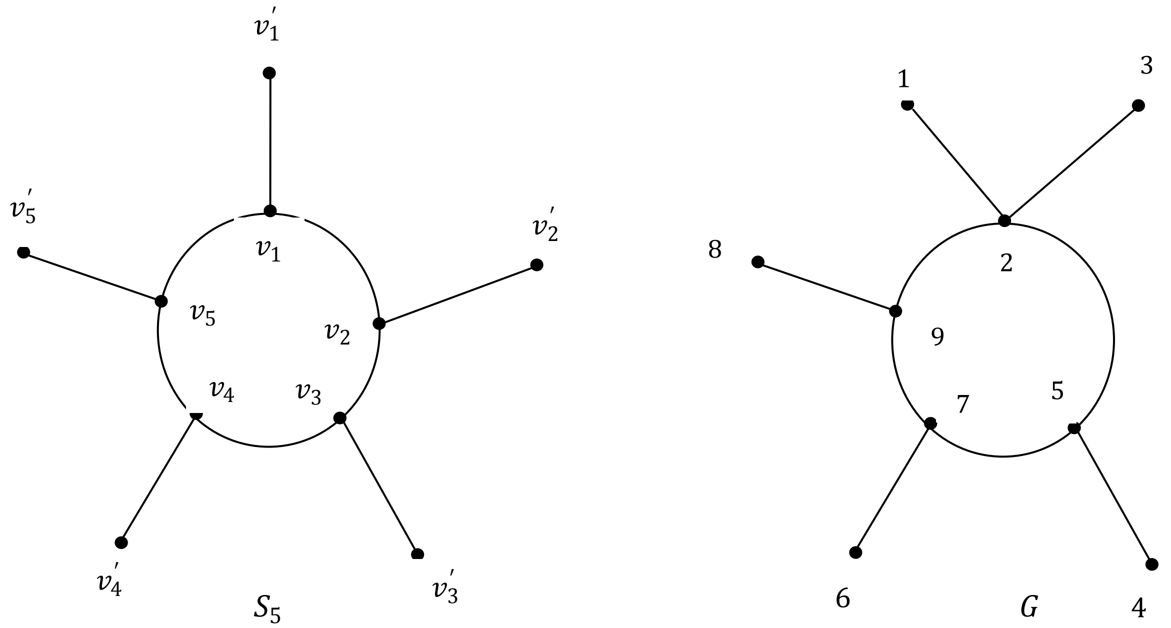


Figure 3: Fusion of two consecutive vertices v_1 and v_2 in S_5.

Proposition 2.3:

The graph obtained by fusing any two non-consecutive vertices in a sunlet graph S_n is a prime graph.

Proof:

Let S_n be a sunlet graph with $2n$ vertices

and let $V(S_n) = \{v_{1,}v_2, \ldots ..v_n, v_1^{'}, v_2^{'}, \ldots .v_n^{'}\}$,

$E(S_n) = \{v_i v_i^{'} / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_1 v_n\}$

Here, $|V(S_n)| = 2n$

Let G be the graph obtained after fusion of two non-consecutive vertices in S_n.

$|V(G)| = 2n - 1$

Case (i):

When fusing any two non-consecutive vertices in $v_i^{'}$ of S_n.

Define a labeling $f: V(G) \rightarrow \{1,2, . .2n - 1\}$ as follows.

Let $f(v_1') = 1$ where v_1 is a new vertex.

$f(v_i) = 2, 3$ where v_i' are the vertices incident with v_i'

$f(v_i) = 2i + 1$ for $2 \leq i \leq n - 1$

$f(v_i') = 2i$ for $2 \leq i \leq n - 1$

Case (ii):

When fusing any two non –consecutive vertices in v_i of S_n.

Define a labeling $f: V(G) \rightarrow \{1,2, \dots \dots \dots . .2n - 1\}$ as follows.

Let $f(v_1) = 1$ where v_1 is a new vertex.

$f(v_i) = 2, 3$ where v_i is incident with v_1

$f(v_i) = 2i + 1$ for $2 \leq i \leq n - 1$

$f(v_i') = 2i$ for $2 \leq i \leq n - 1$

Case (iii):

When fusing any two non-consecutive vertices in one in v_i and other in v_i' of S_n.

Define a labeling $f: V(G) \rightarrow \{1,2, . .2n - 1\}$ as follows.

Let $f(v_1) = 1$ where v_1 is a new vertex

$f(v_i') = 2$ v_i' incident with v_1

$f(v_i) = 3$ v_i is incident with v_1

$f(v_i) = 2i + 1$ for $2 \leq i \leq n - 1$

$f(v_i') = 2i$ for $2 \leq i \leq n - 1$

Then f admits a prime labeling, Thus G is a prime graph.

Example:

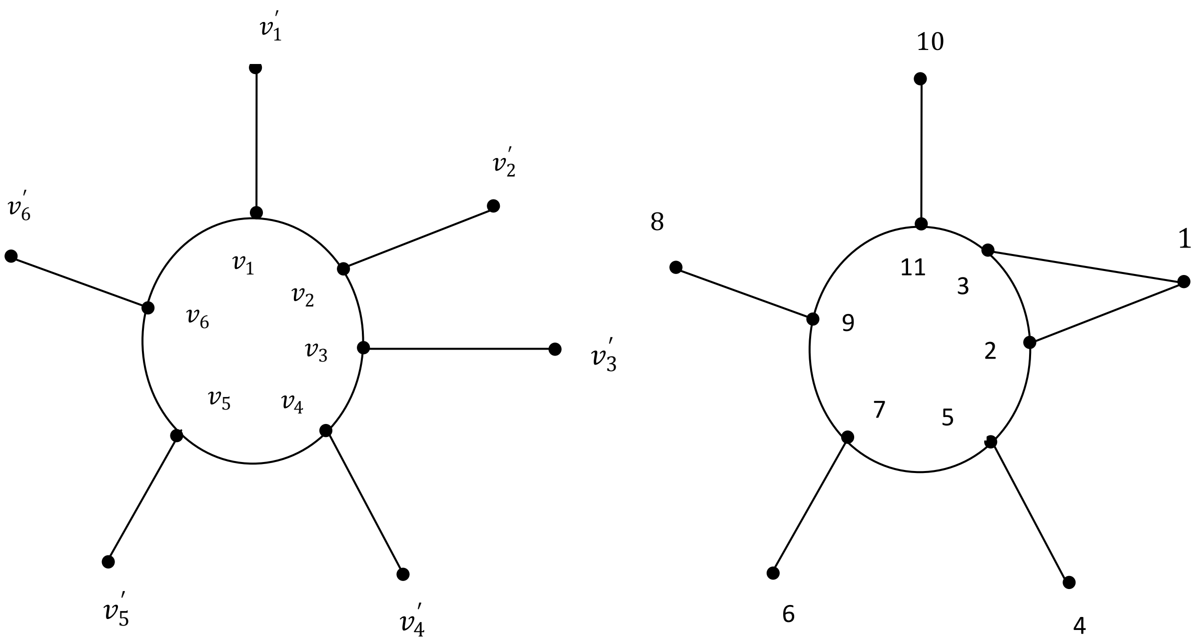


Figure 4: Fusion of two non-consecutive vertices v_1' of v_2' sunlet graph is a prime graph.

Proposition 2.4:

The graph obtained by duplicating a vertex v_k in the rim of Sunlet graph S_n is a prime graph.

Proof:

Let, S_n be a sunlet graph with $2n$ vertices

and $V(S_n) = \{v_1, v_2, \dots . . v_n, v_1', v_2', \dots . v_n'\}$

$$E(S_n) = \{v_i v_i^{'} / \ 1 \le i \le n\} \cup \{v_i v_{i+1} \ /1 \le i \le n-1\} \cup \{v_1 v_n\}$$

Let G_k be the graph obtained by duplicating the vertex v_k in S_n and v_k^* be the new vertex. Then $|V(G_k)| = 2n + 1$.

Define a labeling $f: V(G_k) \to \{1,2,3, ..2n + 1\}$ as follows.

Let $f(v_k) = 1$

$f(v_k^{'}) = 2$

$f(v_k^*) = 3$

$f(v_{k+1}) = 5$

$f(v_{k+1}^{'}) = 4$

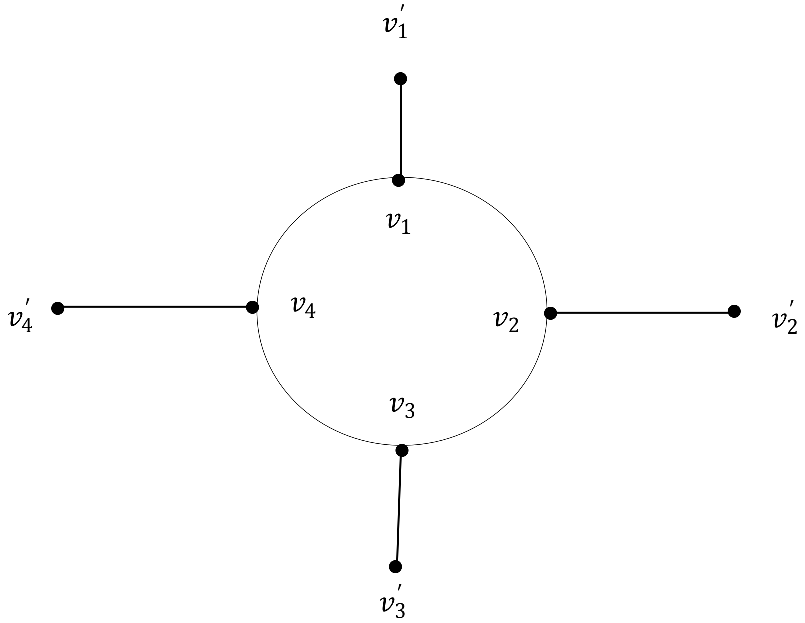
$f(v_i) = 2i + 1$ for $3 \le i \le n$

$f(v_i^{'}) = 2i$ for $3 \le i \le n$

Then f admits a prime labeling.

Thus G_k is a prime graph.

Example:



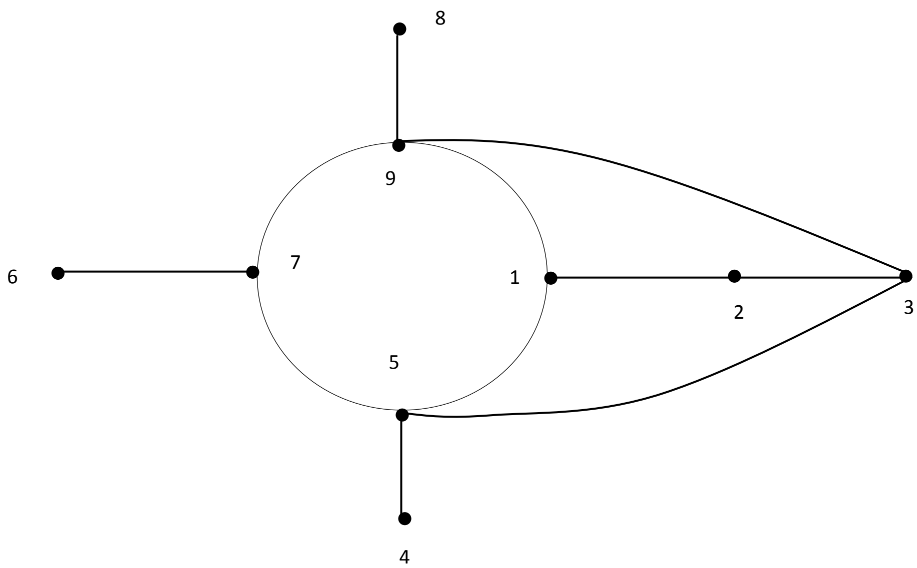


Figure 5: Duplication of v_2 in S_4

Proposition 2.5:

The graph obtained by duplicating a pendant vertex $v_k^{'}$ of Sunlet graph $S_n$is a prime graph.

Proof:

Let S_n be the sunlet graph with $2n$ vertices

Let $V(S_n) = \{v_{1,}v_2, \ldots\ldots v_n, v_1^{'}, v_2^{'}, \ldots.v_n^{'}\}$

$E(S_n) = \{v_i v_i^{'}/ 1 \leq i \leq n\} \cup \{v_i v_{i+1} /1 \leq i \leq n-1\} \cup \{v_1 v_n\}$

Let G_k be the graph obtained by duplicating any pendent the vertex $v_k^{'}$inS_n.

Here $|V(G_k)| = 2n + 1$

Let the new vertex be v_k^*

Define a labeling$f: V(G_k) \to \{1,2,3,..2n+1\}$as follows.

Let $f(v_k) = 1$

$f(v_k^*) = 2$

$f(v_k^{'}) = 3$

$f(v_i) = 2i + 1$ for $2 \leq i \leq n$

$f(v_i^{'}) = 2i$ for $2 \leq i \leq n$

Then f admits a prime labeling .

Thus $G_k$is a prime graph.

Example:

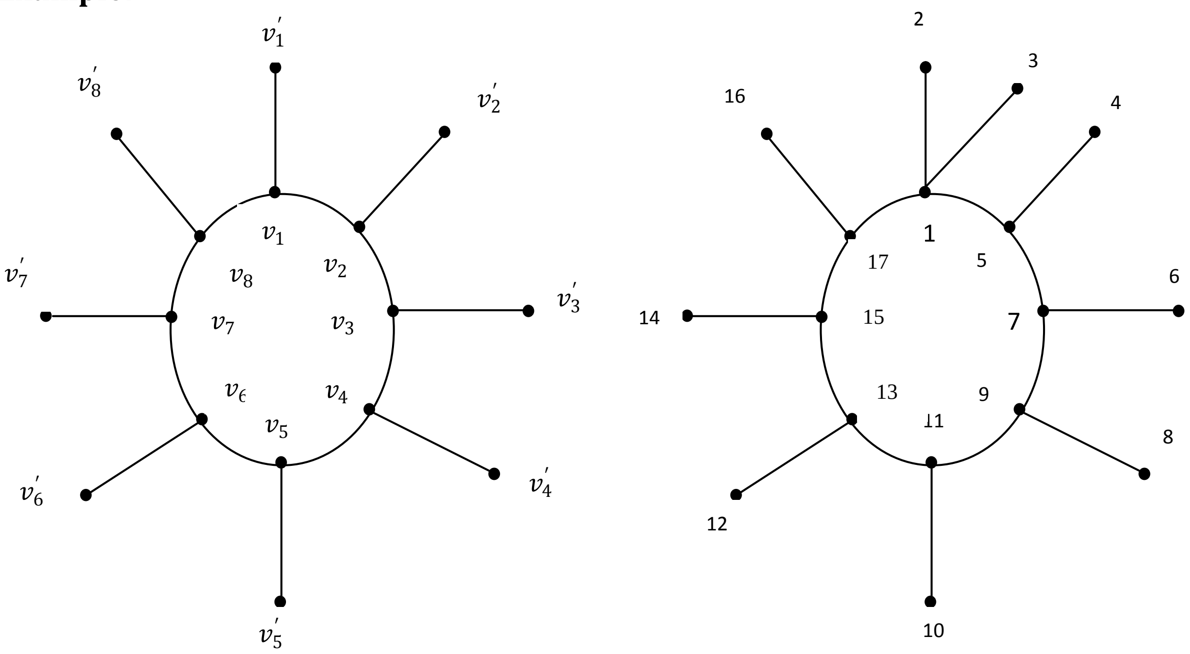


Figure 6: Duplication of a pendant$v_1^{'}$ vertex of sunlet graph S_8

Proposition 2.6:

Let G_k be the graph obtained by switching any vertex v_k in $S_n$then $G_k$is a prime graph.

Proof:

Let S_n be a sunlet graph with $|V(S_n)| = 2n$ and

$V(S_n) = \{v_{1,}v_2, \ldots\ldots v_n, v_1^{'}, v_2^{'}, \ldots.v_n^{'}\}$

$E(S_n) = \{v_i v_i^{'}/ 1 \leq i \leq n\} \cup \{v_i v_{i+1} /1 \leq i \leq n-1\} \cup \{v_1 v_n\}$

Let G_k be the graph obtained by switching the vertex v_k in S_n

Then $|V(G_k)| = 2\text{n}$

Define a labeling $f: V(G_k) \to \{1, 2, 3, ..2n\}$ as follows

Let $f(v_k) = 1$

$f(v_k^{'}) = 2$

$f(v_{k+i}) = 2i + 1$ for $1 \leq i \leq n - k$

$f(v_{k+i}^{'}) = 2i$ for $2 \leq i \leq n - k$

$f(v_i) = 2(n - k) + 2i + 1$ for $1 \leq i \leq k - 1$

$f(v_i^{'}) = 2(n - k) + 2i$ for $1 \leq i \leq k - 1$

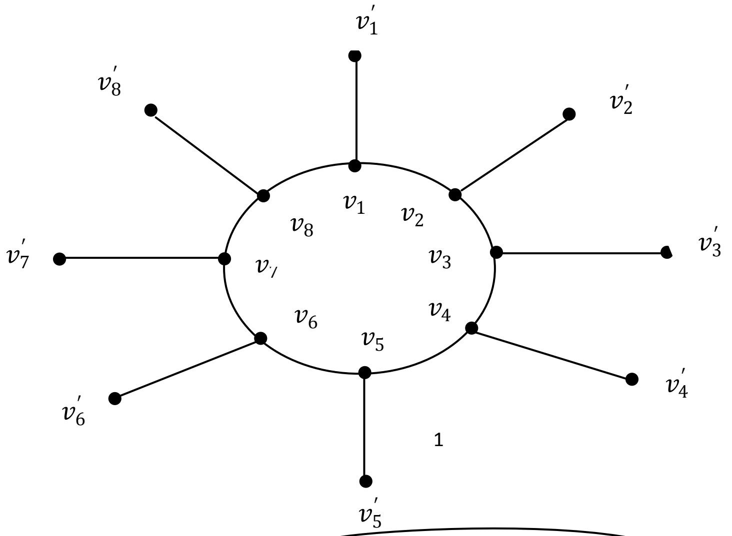
Then for any edge $e = v_i v_j \in G$, $\gcd\left(f(v_i), f(v_j)\right) = 1$

For edge $e = v_i v_i^{'} \in G$, $\gcd\left(f(v_i), f(v_i^{'})\right) = 1$

Then f admits prime labeling

Thus G_k is a prime graph.

Example:



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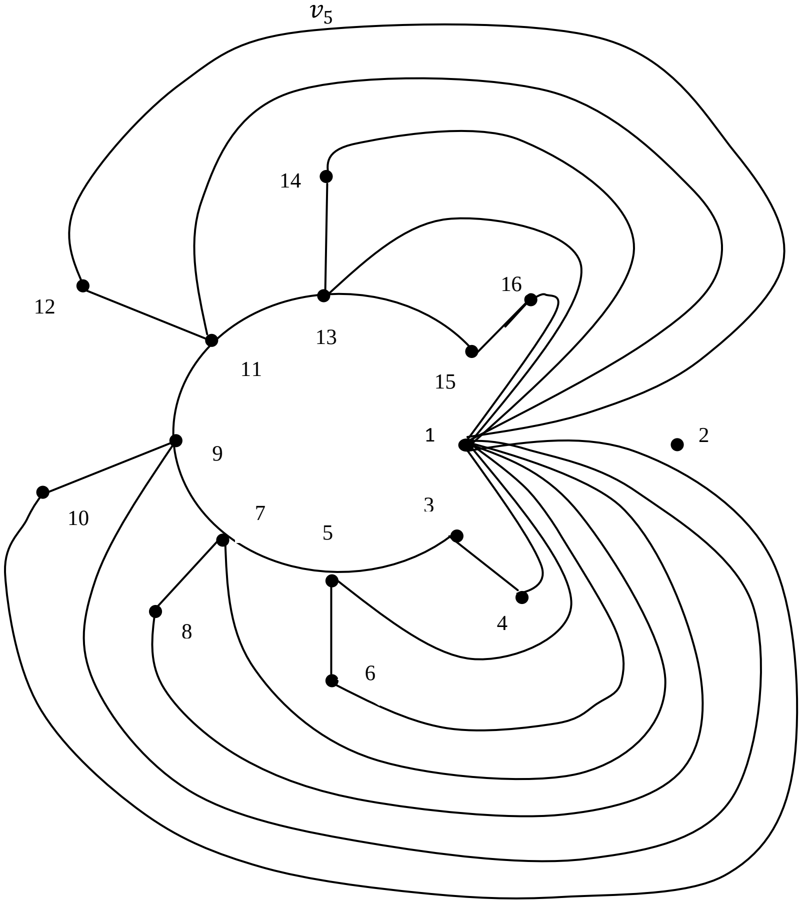

Figure 7: Switching of a vertex v_3 in S_8

Proposition 2.7:

The graph obtained by path union of two pieces Sunlet graph S_n is a prime graph.

Proof:

Consider, two copies of Sunlet graph S_n and S_n^* respectively.

Let $V(S_n) = \{v_{1,}v_2, \ldots..v_n, v_1^{'}, v_2^{'}, \ldots.v_n^{'}\}$

$E(S_n) = \{v_iv_i^{'}/\ 1 \le i \le n\} \cup \{v_iv_{i+1}\ /1 \le i \le n-1\} \cup \{v_1v_n\}$

$V(S_n^*) = \{u_1, u_2, \ldots..u_n, u_1^{'}, u_2^{'}, \ldots\ldots u_n^{'}\}$

$E(S_n^*) = \{u_iu_i^{'}/\ 1 \le i \le n\} \cup \{u_iu_{i+1}\ /1 \le i \le n-1\} \cup \{u_1u_n\}$

Let G_k be the graph obtained by the path union of two pieces of sunlet graphs S_n and S_n^*

$V(G_k) = V(S_n) \cup V(S_n^*)$

$E(G_k) = E(S_n) \cup E(S_n^*) \cup \{v_ku_k\}$

Here $|V(G_k)| = 4n$

Define a labeling $f: V(G_k) \to \{1,2,3, \ldots\ldots\ldots\ldots\ldots.4n\}$ as follows

Let $f(v_1) = 2$

$f(v_1^{'}) = 1$

$f(v_i) = 2i + 1$ for $1 \le i \le n-1$

$f(v_i^{'}) = 2i$ for $1 \le i \le n$

$f(u_i) = 2n + 2i + 1$ for $0 \le i \le n$

$f(u_i^{'}) = 2n + 2i$ for $1 \le i \le n$

Then f admits a prime labeling.

ThusG_k is a prime graph.

Example:

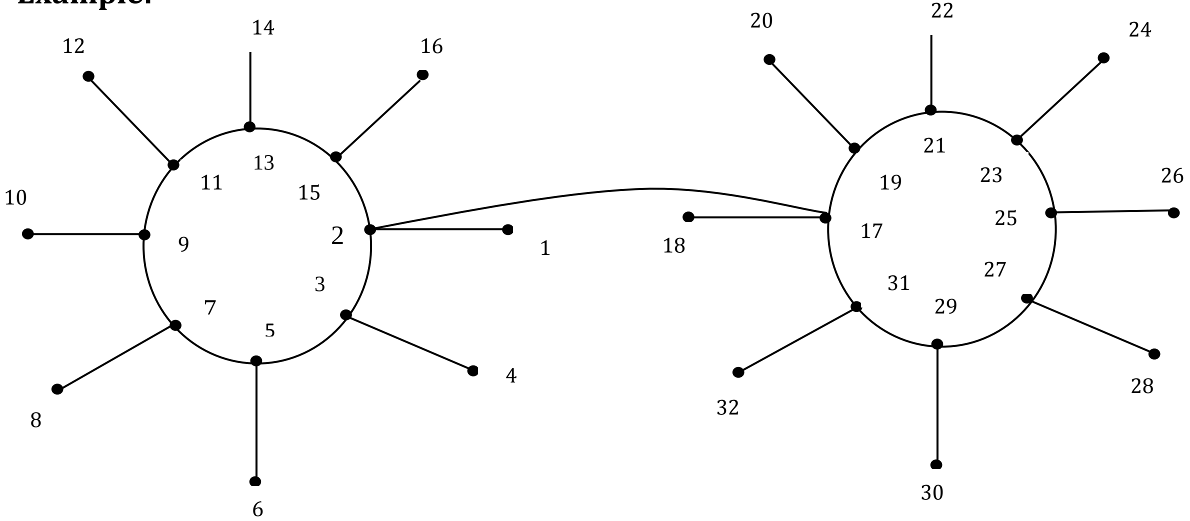

Figure 8: Path union of sunlet graph on 8 vertices and prime labeling

3. Conclusion:

As not every graph admit prime labeling it is very interesting to investigate graph (or) graph families which admit prime graph. In the present paper we investigated a sunlet graph is a prime graph and operation on sunlet graph admits prime labeling and also proved seven new results in this context.

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