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SOFT UNION FUZZY STRUCTURES OF G-MODULES

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Abstract:

In this paper, we introduce fuzzy version of soft uni-G-modules of a vector space with respect to soft structures, which are fuzzy soft uni-G-modules (UFSG-module) .These new concepts show that how a soft set affects on a G-module of a vector space in the mean of intersection, union and inclusion of sets and thus, they can be regarded as a bridge among classical sets, fuzzy soft sets and vector spaces. We then investigate their related properties with respect to soft set operations, soft image, soft pre-image, soft anti image, α-inclusion of fuzzy soft sets and linear transformations of the vector spaces. Furthermore, we give the applications of these new G-module on vector spaces.

Key Words: Soft Set, UFSG-Module, Fuzzy Soft Image, Fuzzy Soft Anti Image, A-Inclusion, Trivial & Whole

1. Introduction:

Most of the problems in economics, engineering, medical science, environments etc. have various uncertainties. We cannot successfully use classical methods to solve these uncertainties because of various uncertainties typical for those problems. Hence some kinds of theories were given like theory of fuzzy sets and rough sets [17], i.e., which we can use as mathematical tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in [15]. Soft set theory was introduced by Molodtsov for modeling vagueness and uncertainty and it has been received much attention since Ali et al [6] and Sezgin and Atagun [34] introduced and studied operations of soft sets. This theory has started to progress in the mean of algebraic structures, since Aktas¸ and Cagman [5] defined and studied soft groups. Since then, soft substructures of rings, fields and modules [8], union soft substructures of near-rings and near-ring modules, normalistic soft groups are defined and studied in detailed. Soft set has also been studied in the following papers [1, 2, 3, 25, 26].The theory of G-modules originated in the 20th century. Representation theory was developed on the basis of embedding a group G in to a linear group GL (V).

The theory of group representation (G module theory) was developed by Frobenious G [1962]. Soon after the concept of fuzzy sets was introduced by Zadeh in 1965. Fuzzy subgroup and its important properties were defined and established by Rosenfeld in 1971. After that in the year 2004 Shery Fernandez introduced fuzzy parallels of the notions of G-modules. This study is of great importance since UFSG-modules show how a fuzzy soft set affect on a G-module of a vector space in the mean of intersection, union and inclusion of sets, so it functions as a bridge among classical sets, soft sets and vector spaces. In this paper, we introduce union fuzzy soft G-modules of a vector space that is abbreviated by UFSG- module and investigate its related properties with respect to fuzzy soft set operations. Then we give the application of fuzzy soft image, fuzzy soft pre image, upper α-inclusion of fuzzy soft sets, linear transformations of vector spaces on vector spaces in the mean of UFSG-modules. Moreover, we apply soft pre image, soft anti-image, lower α-inclusion of soft sets, linear transformations of vector spaces on these fuzzy soft G-modules. The work of this paper is organized as follows. In the second section as preliminaries, we give basic concepts of soft sets and fuzzy soft G-modules. In Section 3, we introduce UFSG-modules and study their characteristic properties. In Section 4, we give the applications of UFSG-modules.

2. Preliminaries:

In this section as a beginning, the concepts of G-module soft sets introduced by Molodsov and the notions of fuzzy soft set introduced by Maji et al. have been presented.

2.1 Definition [36**]:**

Let G be a finite group. A vector space M over a field K (a subfield of C) is called a G-module if for every g ∈ G and m ∈ M, there exists a product (called the right action of G on M) m.g ∈ M which satisfies the following axioms.

1. $m.1_G = m$ for all $m \in M$ (1_G being the identify of G)
2. m. (g. h) = (m.g). h, m ∈ M, g, h ∈ G
3. $(k_1 m_1 + k_2 m_2). G = k_1 (m_1. g) + k_2(m_2. g)$, $k_1, k_2 \in K$, $m_1, m_2 \in M$ & $g \in G$. In a similar manner the left action of G on M can be defined.

2.2. Definition [36**]:**

Let M and M* be G-modules. A mapping Ø: M→M* is a G-module homomorphism if

1. $Ø(k_1 m_1 + k_2 m_2) = k_1 Ø (m_1) + k_2 Ø (m_2)$
2. $Ø(gm) = g Ø (m)$, $k_1, k_2 \in K$, $m, m_1, m_2 \in M$ & $g \in G$.

2.3. Definition [36**]:**

Let M be a G-module. A subspace N of M is a G - sub module if N is also a G-module under the action of G. Let U be a universe set, E be a set of parameters, P(U) be the power set of U and A ⊆ E.

2.4. Definition [29]:

A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11, 17, 18, 26, 29, 7].

2.5. Definition [6]:

The relative complement of the soft set F_A over U is denoted by $F^r{}_A$, where $F^r{}_A$: A → P(U) is a mapping given as $F^r{}_A(a) = U \setminus F_A(a)$, for all a ∈ A.

2.6. Definition [6]:

Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of F_A and G_B is denoted by F_A ⋓ G_B, and is defined as F_A ⋓ G_B =(H,C), where C = A∩B and for all c ∈ C, H(c) = F(c)∩G(c).

2.7. Definition [6]:

Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H,C),where C = A∩B and for all c ∈ C, H(c) = F(c)∪G(c).

2.8. Definition [12]:

Let F_A and G_B be soft sets over the common universe U and ψbe a function from A to B. Then we can define the soft set ψ (F_A) over U, where ψ (F_A) : B→P(U) is a set valued function defined by ψ (F_A)(b) =∪{F(a) | a ∈ A and ψ (a) = b}, if $\psi^{-1}(b) \neq \emptyset$, = 0 otherwise for all b ∈ B. Here, ψ (F_A) is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B)$: A → P(U) is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all a ∈ A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.9. Definition [13]:

Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A) : B \to P(U)$ is a set-valued function defined by $\psi^\star(F_A)(b) = \cap\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq a$, =0 otherwise for all b ∈ B. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

2.1. Theorem [13]:

Let F_H and T_K be soft sets over U, $F^r{}_H$, $T^r{}_K$ be their relative soft sets, respectively and ψ be a function from H to K. then,

i) $\psi^{-1}(T^r{}_K) = (\psi^{-1}(T_K))^r$,

ii) $\psi(F^r{}_H) = (\psi^\star(F_H))^r$ and $\psi^\star(F^r{}_H) = (\psi(F_H))^r$.

2.10. Definition [14]:

Let F_A be a soft set over U and a be a subset of U. Then upper α-inclusion of F_A, denoted by $F_A^{\supseteq\alpha}$, is defined as $F_A^{\supseteq\alpha} = \{x \in A / F(x) \supseteq \alpha\}$. Similarly, $F_A^{\subseteq\alpha} = \{x \in A \mid F(x) \subseteq \alpha\}$ is called the lower α -inclusion of F_A. A nonempty subset U of a vector space V is called a subspace of V if U is a vector space on F. From now on,V denotes a vector space over F and if U is a subspace of V, then it is denoted by U < V.

3. Union Fuzzy Soft G-Modules:

In this section, we first define Union fuzzy soft G-modules of a vector space, abbreviated by UFSG-modules. We then investigate its related properties with respect to soft set operations.

3.1. Definition:

Let G be a group. Let M be a G-module of V and B_M be the fuzzy soft set over V. Then B_M is called Union fuzzy soft G-modules of V, denoted by $B_M \,\widetilde{<}_u\, V$ if the following properties are satisfied: $B(ax+by) \subseteq B(x) \cup B(y)$, $B(\alpha x) \subseteq B(x)$, for all x,y∈ M, and $\alpha \in F$.

3.1. Proposition:

If $B_M \,\widetilde{<}_u\, V$, then $B(0_V) \subseteq B(x)$ for all x ∈ M.

Proof:

Since B_M is an UFSG-module of V, then $B(ax+by) \subseteq B(x) \cup B(y)$ for all x,y∈ M and since (M,+) is a group, if we take by=-ax then, for all x ∈ M,

$B(ax-ax) = B(0_V) \subseteq B(x) \cup B(x) = B(x)$,for all x ∈ M.

In section 3, we showed that restricted intersection, the sum and product of two IFSG-modules of V is an IFSG-modules of V. Now, we show that restricted union of two UFSG-modules of V with the following theorem:

3.1. Theorem:

If $A_{M_1} \,\widetilde{<}_u\, V$ and $B_{M_2} \,\widetilde{<}_u\, V$, then $A_{M_1} \cup_{\mathcal{R}} B_{M_2} \,\widetilde{<}_u\, V$.

Proof:

By definition2.7, let $A_{M_1} \cup_{\mathcal{R}} B_{M_2} = (A, M_1) \cup_{\mathcal{R}} (B, M_2) = (\boldsymbol{Q}, M_1 \cap M_2)$,where $\boldsymbol{Q}(x) = A(x) \cup B(x)$ for all x∈ $M_1 \cap M_2 \neq \emptyset$. Since M_1 & M_2 are G-modules of V, then $M_1 \cap M_2$ is a G-modules of V.

Let x,y ∈ $M_1 \cap M_2$ and $\alpha \in F$.Then

$$\boldsymbol{Q}(ax+by) = A(ax+by) \cup B(ax+by) \subseteq (A(x) \cup A(y)) \cup (B(x) \cup B(y))$$
$$= (A(x) \cup B(x)) \cup (A(y) \cup B(y)) = \boldsymbol{Q}(x) \cup \boldsymbol{Q}(y)$$
$$\boldsymbol{Q}(\alpha x) = A(\alpha x) \cup B(\alpha x) \subseteq A(x) \cup B(x) = \boldsymbol{Q}(x)$$

There fore $A_{M_1} \cup_{\mathcal{R}} B_{M_2} = Q_{M_1 \cap M_2} \,\widetilde{<}_u\, V$.

3.2. Definition:

Let B_M be a UFSG-module of V. Then

(a) B_M is said to be trivial if $B(x) = \{0_V\}$ for all $x \in M$.
(b) B_M is said to be whole if $B(x) = V$ for all $x \in M$.

3.2. Proposition:

Let A_{M_1} and B_{M_2} be UFSG-modules of V, then

(i) If A_{M_1} and B_{M_2} are trivial UFSG-modules of V, then $A_{M_1} \cup_{\mathcal{R}} B_{M_2}$ is a trivial UFSG-module of V.
(ii) If A_{M_1} and B_{M_2} are whole UFSG-modules of V, then $A_{M_1} \cup_{\mathcal{R}} B_{M_2}$ is a whole UFSG-module of V.
(iii) If A_{M_1} is a trivial UFSG-module of V and B_{M_2} is a whole UFSG-modules of V, then $A_{M_1} \cup_{\mathcal{R}} B_{M_2}$ is a whole UFSG-module of V.

Proof:

The proof is easily seen by definition 2.7, definition 3.2, and theorem 3.1.

4. Application of UFSG-Modules:

In this section, first we obtain the relation between IFSG-modules and UFSG-modules and then give the applications of soft pre-image, soft anti-image, lower α-inclusion of soft sets and linear transformation of vector spaces on vector spaces with respect to UFSG-modules.

4.1. Theorem:

Let T_M be a fuzzy soft set over V. Then, T_M is an UFSG-module of V if and only if T_M^r is an IFSG-modules of V.

Proof:

Let T_M be a UFSG-modules of V. Then for all x, y∈ M, a,b, $\alpha \in F$,

$$T_M^r\,(ax+by) = {}^{V}\!/_{T_M(ax+by)} \supseteq {}^{V}\!/_{(T_M(x) \cup T_M(y))}$$
$$= \left({}^{V}\!/_{T(x)}\right) \cap \left({}^{V}\!/_{T(y)}\right) = T_M^r(x) \cap T_M^r(y)$$
$$T_M^r\,(\alpha x) = {}^{V}\!/_{T_M(\alpha x)} \supseteq {}^{V}\!/_{T_M(x)} = T_M^r\,(x)$$

Thus, T_M^r is an IFSG-module of V.

Conversely, let T_M^r be an IFSG-module of V. Then for all x, y∈ M, and $\alpha \in F$,

$$T_M(ax+by) = {}^{V}\!/_{T_M^r(x+y)} \subseteq {}^{V}\!/_{(T^r(x) \cap T^r(y))}$$
$$= \left({}^{V}\!/_{T^r(x)}\right) \cup \left({}^{V}\!/_{T^r(y)}\right) = T(x) \cup T(y)$$
$$T_M(\alpha x) = {}^{V}\!/_{T_M^r(\alpha x)} \subseteq {}^{V}\!/_{T^r(\alpha x)} = T(x)$$

Thus, T_M is an UFSG-module of V.

The above theorem shows that if a fuzzy soft set is a UFSG-module of V, then its relative complement is an IFSG-module of V and vice versa.

4.2. Theorem:

If $T_M \,\widetilde{<}_u\, V$, then $M_T=\{x \in M / \; T(x)=T(0_V)\}$ is a G- module of M.

Proof:

It is seen that $0_V \in M_T$ and $\emptyset \neq M_T \subseteq M$. We need to show that ax+by $\in M_T$ and $\alpha x \in M_T$, for all x,y $\in M_T$ and $\alpha \in F$. Since x,y $\in M_T$ and T_M is a UFSG- module of V, then $T(x)=T(y)= T(0_V)$, $T(ax+by) \subseteq T(x) \cup T(y) \subseteq T(0_V)$.$T(\alpha x) \subseteq T(x) = T(0_V)$ for all x,y $\in M_T$ and $\alpha \in F$. Furthermore, $T(0_V) \subseteq T(ax+by)$ and $T(0_V) \subset T(\alpha x)$.Thus, the proof is completed.

4.3. Theorem:

Let G_M be a fuzzy soft set over V and α be a subset of V such that $\alpha \supseteq G(0_V)$. If G_M is a UFSG-module of V , then $G_M^{\subseteq\alpha}$ is a G-module of V.

Proof:

Since $\alpha \supseteq G(0_V)$,then $0_V \in G_M^{\subseteq\alpha}$ and $\emptyset \neq G_M^{\subseteq\alpha} \subseteq V$.
Let x,y $\in G_M^{\subseteq\alpha}$ and a,b, $\alpha \in$ F,then G(x)$\subseteq\alpha$ and G(y)$\subseteq\alpha$.
We need to show that ax+by $\in G_M^{\subseteq\alpha}$ and nx$\in G_M^{\subseteq\alpha}$ for all x,y $\in G_M^{\subseteq\alpha}$ and n$\in$ F. Since G_M is a UFSG –module of V, it follows that G(ax+by)$\subseteq$G(x) $\cup$ G(y)$\subseteq \alpha \cup \alpha = \alpha$
Therefore, G(nx)$\subseteq$G(x)$\subseteq \alpha$, which completes the proof.

4.4. Theorem:

Let G_M and T_W be a fuzzy soft set over V, where M and W are G-modules of V and Ψ be a linear transformation from M to W. If T_W is a UFSG-module of V, then so is $\Psi^{-1}(T_W)$.

Proof:

Let T_W be a UFSG-module of V. Then ${T_W}^r$ is an IFSG- module of V by theorem:6.1 and $\Psi^{-1}({T_W}^r)$ is an IFSG- module of V by theorem4.4. Thus, $\Psi^{-1}({T_W}^r)$ =$(\Psi^{-1}(T_W))^r$ is an IFSG- module of V by theorem 2.1. Therefore, $\Psi^{-1}(T_W)$ is a UFSG –module of V by theorem 4.1.

4.5. Theorem:

Let G_M and T_W be a fuzzy soft sets over V, where M and W are G- Modules of V and Ψ be a linear isomorphism from M to W. If G_M is a UFSG- Module of V, then so is $\Psi^*(G_M)$

Proof:

Let G_M be a UFSG- module of V. Then ${G_M}^r$ is an IFSG- module of V by theorem: 6.1 and $\Psi^{({G_M}^r)}$ is an IFSG- module of V by theorem4.3. Thus, $\Psi^{({G_M}^r)} = (\Psi^*(G_M))^r$ is an IFSG-module of V by theorem 2.1. So, $\Psi^*(G_M)$ is a UFSG- module of V by theorem4.1.

4.6. Theorem:

Let V_1 and V_2 be two vector spaces and $(T_1, M_1) \tilde{<}_u V_1$, $(T_2, M_2) \tilde{<}_u V_2$.
If $f: M_1 \to M_2$ is a linear transformation of vector spaces, then

(i)If f is surjective, then $(T_1, f^{-1}(M_2)) \tilde{<}_u V_1$,
(ii)$(T_2, f(M_1)) \tilde{<}_u V_2$,
(iii)$(T_1, \mathrm{kerl} f) \tilde{<}_u V_1$.

Proof:

Follows from theorem 3.1 and theorem 4.5 .Therefore omitted.

4.1. Corollary:

Let $(T_1, M_1) \tilde{<}_u V_1$, $(T_2, M_2) \tilde{<}_u V_2$. If $f: M_1 \to M_2$ is a linear transformation, then $(T_2, \{0_{M_2}\}) \tilde{<}_u V_2$.

Proof:

Follows from definition 4.6 (ii) and theorem 4.6 (iii).

Conclusion:

Throughout this paper, we have dealt with UFSG-modules and UFSG-modules of a vector space. We have investigated their related properties with respect to soft set operations obtained the relations on UFSG-modules. Furthermore, we have derived some applications of UFSG-modules with respect to soft image, soft pre image, soft anti image, α-inclusion of soft sets and linear transformations of vector spaces. Further study could be done for fuzzy soft sub structures of different algebras.

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