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BIO MATHEMATICAL MODEL TO FIND THE GALLBLADDER CONTRACTION OUTCOMES USING NORMAL DISTRIBUTION

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Abstract:

The cholecystokinin (CCK) is a 33 amino acid polypeptide. The C-terminal octapeptide fragment (OP-CCK) reproduces all the known biologic activities of the intact molecule. The purpose of this study was to determine the effect on gallbladder contraction of 20ng/kg of OP-CCK intravenously in patients undergoing cholecystography. From the system with Poisson input flows the generating function of the unique stationary queue length distribution, (i.e) in the $M^X/M^R/1/\infty$ queueing system is used to find the effect of gallbladder contraction outcomes using normal distribution.

Key Words: Gallbladder, Cholecystokinin, $M^X/M^R/1/\infty$ Queueing System & Normal Distribution

1. Introduction:

The diagnostic value of gallbladder contraction during cholecystography is uncertain possible advantages include: visualization of gallstones not seen without contraction; visualization of bile ducts and choledocholithiasis; reproduction of the spontaneous pain of biliary dyskinesia, and correlation of the pain with roentgen graphic features, and possible prediction of future cholelithiasis [8-9]. The common bile duct was visualized in 47 percent of our patients after OP-CCK, Gallbladder contraction occurs at a more rapid rate after OP-CCK than after a fat meal, use of OP-CCK can shorten the time and probably decrease the number of film exposures and amount of radiation required to perform oral cholecystography [5] & [11]. We study in this paper, single channel queueing system with renovation. The solutions for the queues with Poisson input flow are discussed. In the $M^X/M^R/1/\infty$ queueing system, the generating function of the ordinary Poisson is given by $\pi(d) = \frac{1-d^*}{d-d^*}$ where d^* is the root of the equation $\lambda d^2 - (\lambda + \mu)d + p\mu = 0$ is used to evaluate the effect of gallbladder contraction outcomes using normal distribution.

2. Mathematical Model:

Consider a queueing system with renovation. We assume that the system has only one queue with FIFO discipline and one input flow of customers. The input flow is assumed to be Poisson. Denote by λ the input flow intensity. Service times are mutually independent and exponentially distributed random variables with parameter μ. After a customer is serviced, all others can leave the queue with probability q. Only a customer who receives service leaves the system with probability $p = 1 - q$. Below we consider two classes of queues with renovation, Poisson input flow, and exponential service time distribution [1-4].

2.1 $M/M^R/N$ Queueing System:

Denote by N the size of the waiting room. Let ξ_t be the number of customers in the system at moment t. Then $\xi_t \le N$. Denote $\pi_j(t) = Pr\{\xi_t = j\}, j = 0, 1, \dots, N$. The stationary distribution of the queue length is given by the following assertion.

2.2 Proposition:

If $0 < p < 1$ and $\mu > 0$, then for each initial state of the queue satisfying the condition $\pi_j = \lim_{t\to\infty} Pr\{\xi_t = j\}, j = 0, 1, \dots, N$. The generating function of this distribution satisfies the equation

$$\pi(d) = \frac{\mu p\,(1-d)\pi_0 - q\mu d - \lambda\pi_N(1-d)d^{N+1}}{Q(d)}$$

where $Q(d) = \lambda d^2 - (\lambda+\mu)d + p\mu$, and the probabilities π_0 and π_N are given by the formulas $\pi_0 = 1 - \frac{d^{*N} - d_*^N}{d^{*N+1} - d_*^{N+1}}$ and $p_N = \frac{d^* - d_*}{d^{*N+1} - d_*^{N+1}}$ with d^* and d_* being the roots of the equation $Q(d) = O$.

Proof:

The process ξ_t is a Markov chain. It is easy to verify that for $0 < p < 1$ and $\mu > 0$ this Markov chain is ergodic. The stationary distribution $\{\pi_j\}_{j=0}^{\infty}$ of ξ_t satisfies the equations

$$(\lambda+\mu)\pi_k = \lambda p_{k-1} + \mu p\pi_{k+1}, k = 1, 2, \dots, N-1$$
$$\lambda\pi_{N-1} = \mu\pi_N \qquad (1)$$
$$\lambda\pi_0 = \mu\pi_1 + \mu q \sum_{j=2}^{N}\pi_j$$

and the normalization condition $\sum_{j=0}^{N}\pi_j = 1$

Then, from (1) after standard calculations we obtain

$$\pi(z)Q(d) = \mu p(1-d)\pi_0 - q\mu d - \lambda\pi_N(1-d)d^{N+1}$$

The equation $Q(d) = 0$ has two roots d^* and d_*. Both roots are real and positive. The generating function $\pi(d)$ is a polynomial. The stationary probabilities π_0 and π_N must satisfy the equations $\mu p\pi_0(1-d_*) - \lambda\pi_N d_*^{N+1}(1-d_*) = \mu q d_*$ (2)

$$\mu p\pi_0(1-d^*) - \lambda\pi_N d^{*N+1}(1-d^*) = \mu q d^* \qquad (3)$$

From (2) and (3), we obtain the relations for π_0 and π_N. Proposition 2.2 is proved.

2.3 Corollary:

If the renovation probability $q = O$, then the stationary distribution of the queue length is given by $\pi_k = \pi_0 p^{k-1}, k = 1, 2, \dots, N$ and $\pi_0 = \frac{1-\rho}{1-\rho^N}$ where $\rho = {}^{\lambda}/_{\mu}$. If the renovation probability $q = 1$, then the stationary distribution of the queue length is given by

$$\pi_k = \frac{\mu}{\lambda+\mu}\left(\frac{\lambda}{\lambda+\mu}\right)^k, k = 0, 1, 2, \dots, N-1 \text{ and } \pi_N = \left(\frac{\mu}{\lambda+\mu}\right)^N$$

Proof:

The first part of the corollary is well known [1] & [6]. The second part follows directly from the equations (1) for the equilibrium distribution of the queue length process (See [10]).

2.4 $M^X/M^R/1/\infty$ Queueing System:

Let us study now the case $N = \infty$. We consider a more general situation of bulk input. Assume that the customers arrive in groups of random size, and interarrival times are mutually independent, exponentially distributed random variables with parameter λ. This arrival flow is usually called a compound Poisson process. Denote

$$p_k = Pr\{l = k\}, k = 1, 2, \dots$$

where l is the size of an arrival group of customers. We denote by $p(d)$ the generating function of this distribution. Thus, $p(d) = \sum_{k=0}^{\infty} p_k d^k$ where $|d| \le 1$. The average size of the arriving groups is assumed to be finite. Then the function $p(d)$ is regular in the unit disc $D = \{d: |d| \leq 1\}$ and the average size of the arrival group is equal to $l = p'(1)$ (See [6-7]).

2.5 Proposition:

If $0 < p < 1$, $El < \infty$ and $\mu > 0$, then for each initial state, the queueing system has a unique stationary queue length distribution. The generating function of this distribution is given by the formula $\pi(d) = \frac{\mu p(1-d)\pi_0 - q\mu d}{Q_p(d)}$, where

$$Q_p(d) = \lambda dp(d) - (\lambda + \mu)d + p\mu, \pi_0 = \frac{qd_*}{p(1-d_*)}$$

and d_* is a unique root of the equation $Q_p(d) = 0$, that satisfies the inequality $0 < d_* < l$.

Proof:

Denote by $q_m, (m = 1, 2, 3, \ldots)$the number of customers in the system at the arrival epoch τ_m of the m[th] customer. Using the standard technique it is not difficult to verify that $\{q_m\}$ is a positive supermartingale. On the other hand, the queue length ξ_t at the moment t, $\tau_m < t \leq \tau_{m+1}$ satisfies the inequality $0 \leq \xi_t \leq q_m$ for $m \geq 0$. The last inequality implies that the process t has a stationary distribution $\pi_n = \lim_{t\to\infty} Pr\{\xi_t = n$ that satisfies the equations

$$(\lambda + \mu)\pi_k = \lambda \sum_{n=1}^{\infty} \pi_{k-n} p_n + \mu p \pi_{k+1}, k = 1, 2, \ldots \quad (4)$$

$$\lambda\pi_0 = \mu\pi_1 + \mu q \sum_{j=2}^{N} \pi_j \quad (5)$$

Then from (4) and (5) we have

$$\pi(d) = \mu \frac{p(1-d)\pi_0 - qd}{\lambda dp(d) - (\lambda+\mu)d + p\mu} \quad (6)$$

The generating function $\pi(d)$ is regular in the unit disc D. By Rouche's theorem, the equation $\lambda dp(d) - (\lambda + \mu)d + p\mu = 0$ has only one root, $d_* \in D$. Therefore, the numerator of the fraction in (6) should be equal to 0 at $d = d_*$. Hence, $\pi_0 = \frac{qd_*}{p(1-d_*)}$.

Therefore, the generating function of the stationary distribution satisfies the relation

$$\pi(d) = \mu q \frac{d - d_*}{d_* - 1} \frac{1}{Q_p(d)}$$

2.6 Corollary:

If the input flow of customers is ordinary Poisson $(p(d) = d)$, then

$$\pi(d) = \frac{1 - d^*}{d - d^*} \quad (7)$$

where d^* is the root of the equation $\lambda d^2 - (\lambda + \mu)d + p\mu = 0$, satisfying the inequality $z^* > 1$.

3. Example:

The effect of an intravenous injection of 20ng/kg of the C-terminal octapeptide of cholecystokinin (OP-CCK) on gallbladder contraction was investigated in 30 adult patients undergoing oral cholecystography. The figure shows the mean reduction in gallbladder size, 5, 10 and 15 mintues after injection of 20ng/kg of OP-CCK. The mean peak reduction in gallbladder size was 48 percent [11]. Less than 40% decrease in gallbladder size occurred in 4 patients. An additional injection of 40ng/kg of OP-CCK in these 4 patients produced further contraction in only one patients produced further contraction in only one patient. The common bile duct was visualized in 47% of patients after 20ng/kg of OP-CCK. This figure is comparable to that obtain with a standard fat meal [5] & [8-9].

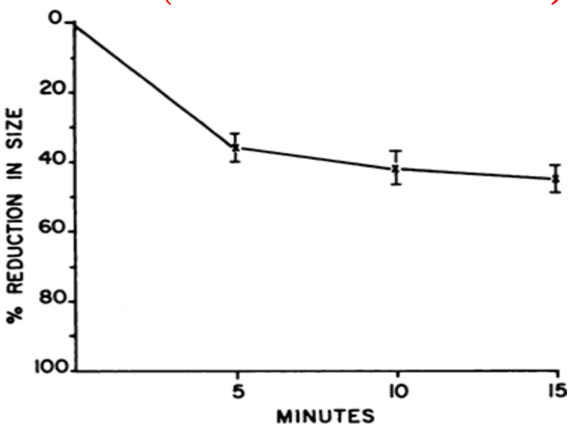

Figure (1): Mean reduction in gallbladder size at 5, 10 and 15 mintues after intravenous injection of 20ng/kg

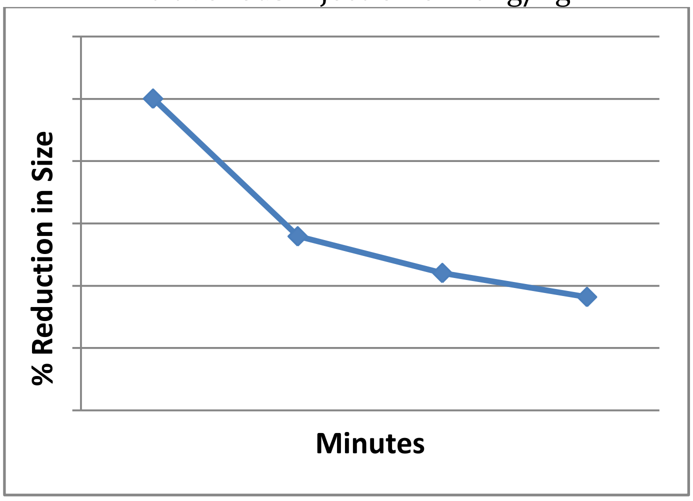

Figure (2): Mean reduction in gallbladder size at 5, 10 and 15 mintues after intravenous injection of 20ng/kg (Using Normal Distribution)

4. Conclusion:

The $M^X/M^R/1/\infty$ queueing system with renovation gives same result as the medical report mentioned above. By using normal distribution (ND) the mathematical model gives the result as same as the medical report. The medical reports {Figure (1)} are beautifully fitted with the mathematical model {Figure (2)}; $(i.e)$ the results coincide with the mathematical and medical report.

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