International Journal of Scientific Research and Modern Education (IJSRME)
ISSN (Online): 2455 – 5630
(www.rdmodernresearch.com) Volume I, Issue I, 2016

MATHEMATICAL EXPRESSION OF RADIATIVE RADIAL FINS WITH TEMPERATURE - DEPENDENT THERMAL CONDUCTIVITY

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Abstract:

In this study, the mathematical expression of radiative radial fins with temperature-dependent thermal conductivity is presented. The approximate analytical solutions of the dimensionless temperature and fin efficiency are derived by using the Homotopy analysis method. Our analytical results are compared with the previous work (DTM solution) and a satisfactory agreement is noted. The Homotopy analysis method can be easily extended to solve other non-linear initial and boundary value problems in physical and chemical sciences.

Key Words: Radial Fin, Fin Efficiency, Variable Thermal Conductivity, Homotopy Analysis Method & Non-Linear Heat Transfer Equation.

1. Introduction:

Extended surfaces (also known as fins) are used to augment heat dissipation from a hot surface through its radiative radial surfaces. In particular, fins are extensively used in various industrial applications such as the cooling of computer processors, air conditioning and oil carrying pipe lines. Several studies were performed on heat transfer using fins. The effects of temperature–dependent thermal conductivity of a moving fin and added radiative component to the surface heat loss have been studied by Khani et.al [6].They applied the Homotopy Analysis method(HAM) to solve governing equations. Arslanturk [4] obtained efficiency and fin temperature distribution by Adomain decomposition method (ADM) and the Homotopy perturbation method (HPM) with temperature dependent thermal conductivity. Using the temperature distribution, the efficiency of the fin is expressed through a term called thermo-geometric parameter (ψ) and thermal conductivity parameter (β), describing the variation of the thermal conductivity. All these problems and phenomena are modified by ordinary or partial differential equations. In most cases, these problems do not admit analytical solution to these equations should be solved using special techniques. Integral transform methods such as the Laplace and the Fourier transform methods are widely used in engineering problems. These methods are more complex and difficult when applying to nonlinear problems. Perturbation methods depend on a small parameter which is difficult to be found for real life non-linear problems. Parameterized perturbation method (PPM) helps us to overcome this shortcoming and improves the accuracy of solution. The purpose of this study is obtaining an analytical solution for temperature distribution of a fin with temperature–dependent thermal conductivity, heat transfer coefficient and heat generation.

2. Mathematical Formulation of the Problem:

A typical heat pipe/fin space radiator is shown in Fig. 1. Both surfaces of the fin are radiating to the outer space at a very low temperature, which is assumed equal to zero absolute. The fin has temperature-dependent thermal conductivity k, which depends on temperature linearly, and fin is diffuse-grey with emissivity ε. The tube surfaces' temperature and the base temperature T_b of the fin are constant, and the convective exchange between the fin and the heat is neglected. The temperature distribution within the fin is assumed to be one dimensional, because the fin is assumed to be thin. Hence, only fin tip length b is considered as the computational domain.

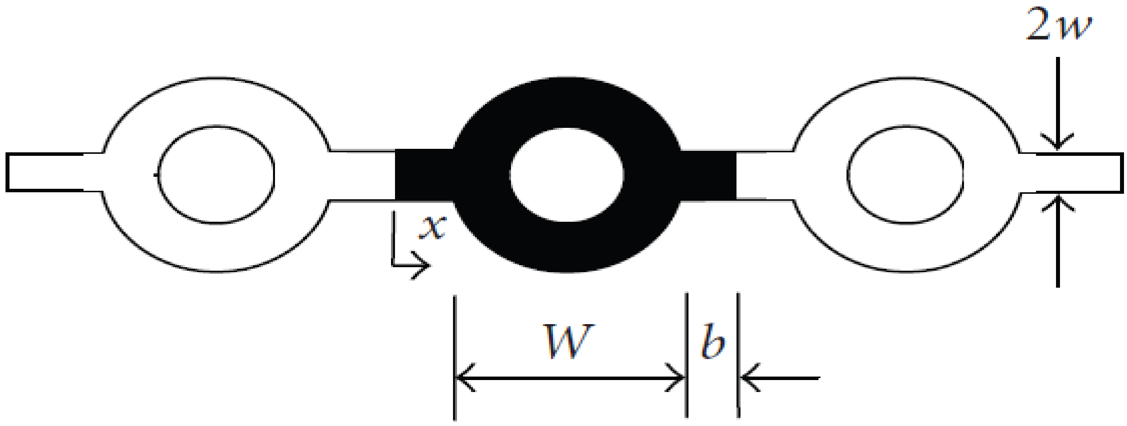

Fig. 1: A heat pipe/fin radiating element

The energy balance equation for a differential element of the fin is given by

$$2w\frac{d}{dx}[k(T)\frac{dT}{dx}] - 2\varepsilon\sigma(T^4 - T_s^4) + q = 0 \qquad (1)$$

Where and σ are thermal conductivity and the Stefan-Boltzmann constant, respectively.

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$k(T) = k_0[1 + \lambda(T - T_a)] \qquad (2)$$

where k_0 is the thermal conductivity at the T_a temperature of the fin and λ is the measure of variation of the thermal conductivity with temperature.

We introduce the following dimensionless quantities as

$$\theta = \frac{T}{T_b},\ \theta_a = \frac{T_a}{T_b},\ \theta_s = \frac{T_s}{T_b},\ \xi = \frac{x}{b},\ \beta = \lambda T_b,\ \psi = \frac{\varepsilon\sigma b^2 T_b^3}{k_0 w},\ Q = \frac{b^2 q}{T_b k_0} \qquad (3)$$

The formulation of the fin problem reduces to the following equation:

$$\frac{d}{d\xi}[(1 + \beta(\theta - \theta_a))\frac{d\theta}{d\xi}] - \psi(\theta^4 - \theta_s^4) + Q = 0, \qquad 0 \le \xi \le 1 \qquad (4)$$

The corresponding boundary conditions are as follows:

$$\frac{d\theta}{d\xi} = 0 \quad at \quad \xi = 0 \qquad (5)$$

$$\theta = 1 \quad at \quad \xi = 1 \qquad (6)$$

3. Fin Efficiency:

The heat transfer from the fin is found by using Newton's law of cooling:

$$Q = \int_0^b P(T - T_a)dx \qquad (7)$$

The ratio of energy radiated away by the fin to the energy that would be radiated if the entire fin were at the base temperature is called the fin efficiency:

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^b P(T - T_a)dx.}{Pb(T_b - T_a)} = \int_{\xi=0}^{1} \theta(\xi)d\xi \qquad (8)$$

4. Solution of the Problems Using the Homotopy Analysis Method:

HAM is a non-perturbative analytical method for obtaining series solutions to nonlinear equations and has been successfully applied to numerous problems in science and engineering [7-22]. In comparison with other perturbative and non-perturbative analytical methods, HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter. Because of this, HAM has proved to be the most effective method for obtaining analytical solutions to highly non-linear differential equations. Previous applications of HAM have mainly focused on non-linear differential equations in which the non-linearity is a polynomial in terms of the unknown function and its derivatives. As seen above, the non-linearity present in electro hydrodynamic flow takes the form of a rational function, and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, HAM yields excellent results.

Liao [7-15] proposed a powerful analytical method for non-linear problems, namely the Homotopy analysis method. This method provides an analytical solution in terms of an infinite power series. However, there is a practical need to evaluate this solution and to obtain numerical values from the infinite power series. In order to investigate the accuracy of the Homotopy analysis method (HAM) solution with a finite number of terms, the system of differential equations were solved. The Homotopy analysis method is a good technique comparing to another perturbation method. The Homotopy analysis method contains the auxiliary parameter h, which provides us with a simple way to adjust and control the convergence region of solution series. The approximate analytical solution of the eqns. (4)-(6) using the Homotopy analysis method is

$$\theta(\xi) == \left\{ \begin{array}{l} \dfrac{s\xi^2}{2u} + 1 - \dfrac{s}{2u} + \dfrac{-h}{2u^2}\left\{ \begin{array}{l} -s\beta\left[s\left(\dfrac{\xi^4}{4} - \dfrac{\xi^2}{2}\right) + u\xi^2 \right] \\ + \dfrac{\psi}{8u^3}\left[\begin{array}{l} s^4\left(\dfrac{\xi^6}{30} - \dfrac{\xi^5}{5} + \dfrac{\xi^4}{2} - \dfrac{2\xi^3}{3} + \dfrac{\xi^2}{2}\right) \\ + 8s^3u\left(\dfrac{\xi^5}{20} - \dfrac{\xi^4}{4} + \dfrac{\xi^3}{2} - \dfrac{\xi^2}{2}\right) \\ + 24s^2u^2\left(\dfrac{\xi^4}{12} - \dfrac{\xi^3}{3} + \dfrac{\xi^2}{2}\right) \\ + 32su^3\left(\dfrac{\xi^3}{6} - \dfrac{\xi^2}{2}\right) + 8u^4\xi^2 \end{array} \right] \end{array} \right\} \\ -\dfrac{1}{2u^2}\left\{ -s\beta\left[\dfrac{s}{4}(-1 + 4u) \right] + \dfrac{\psi}{8u^3}\left[\dfrac{s^4}{2} - \dfrac{8s^3u}{5} + 6s^2u^2 - \dfrac{32su^3}{3} + 8u^4 \right] \right\} \end{array} \right\} \qquad (9)$$

Where

$$s = -Q - \psi\theta_s^4 \quad (10)$$

$$u = 1 - \beta\theta_a \quad (11)$$

5. Results and Discussion:

In this section the effects of physical parameters including thermal conductivity parameter β, radiation-conduction parameter ψ, dimensionless volumetric heat generation Q and dimensionless radiation sink temperature θ_a on temperature distribution will be presented. Fig. 1 illustrates the schematic diagram of a heat pipe/fin radiating element. Fig. 2 to 7 shows the dimensionless temperature $\theta(\xi)$ w.r.to the dimensionless axial distance ξ.

From Fig. 2, we infer that when the radiation-conduction parameter ψ increases, the dimensionless temperature θ decreases for different values of thermal conductivity β and for some fixed values of other parameter θ_a, θ_s, Q, where the HAM and the DTM results are completely coincident. From Fig.3we depicts that the dimensionless temperature distribution along the fin surface with β varying from – 0.3 to 0.3. The curve marked $\beta = 0$ denotes the case when the thermal conductivity is a constant, the curves with $\beta > 0$ correspond to fin materials whose thermal conductivity increases as temperature increases. The converse is true of curves with $\beta < 0$ .From Fig. 4we illustrate the effect of the radiation-conduction parameter ψ, on the temperature distribution in the fin. As ψ increases, the corresponding dimensionless temperature decreases for different values of ψ and for some fixed values of the other parameter $\theta_a, \theta_s, \beta, Q$.

From Fig. 5 we observed that as θ_a increases, the dimensionless temperature within the fin decreases for different values of θ_a and for some fixed values of other parameter θ_s, β, Q, ψ. Fig. 6 illustrate as θ_s increases, the dimensionless temperature within the fin also increases for different values of the radiation sink temperature θ_s and for some fixed values of other parameter θ_a, β, Q, ψ .Fig. 7 shows that an increase in the value of Q causes an increase in the value of θ within the fin for different values of the dimensionless heat generation Q and for some fixed values of other parameter $\theta_a, \theta_s, \beta, \psi$.

Fig. 8 to 11 represents the fin efficiency (η) w.r.to the radiation-conduction fin parameter (ψ). These Figs. clearly demonstrates that the increase in the values of radiation-conduction parameter produces a decrease in the value of the fin efficiency and it reaches the steady state at $\psi = 10$. Fig. 8 shows that the efficiency of the fin decreases for various values of β and in some fixed values of other dimensionless parameters $\theta_a, \theta_s, \beta, Q$. Fig. 9 represents the decrease in the value of fin efficiency for various values of Q and for some fixed values of other parameter $\theta_a, \theta_s, \beta, \psi$. Similarly Fig. 10 and 11 represents the decrease in the value of the fin for various values of θ_a *and* θ_s respectively and for some fixed values of the other parameter. Table 1 illustrates comparison of the results of HAM solution and the DTM solution.

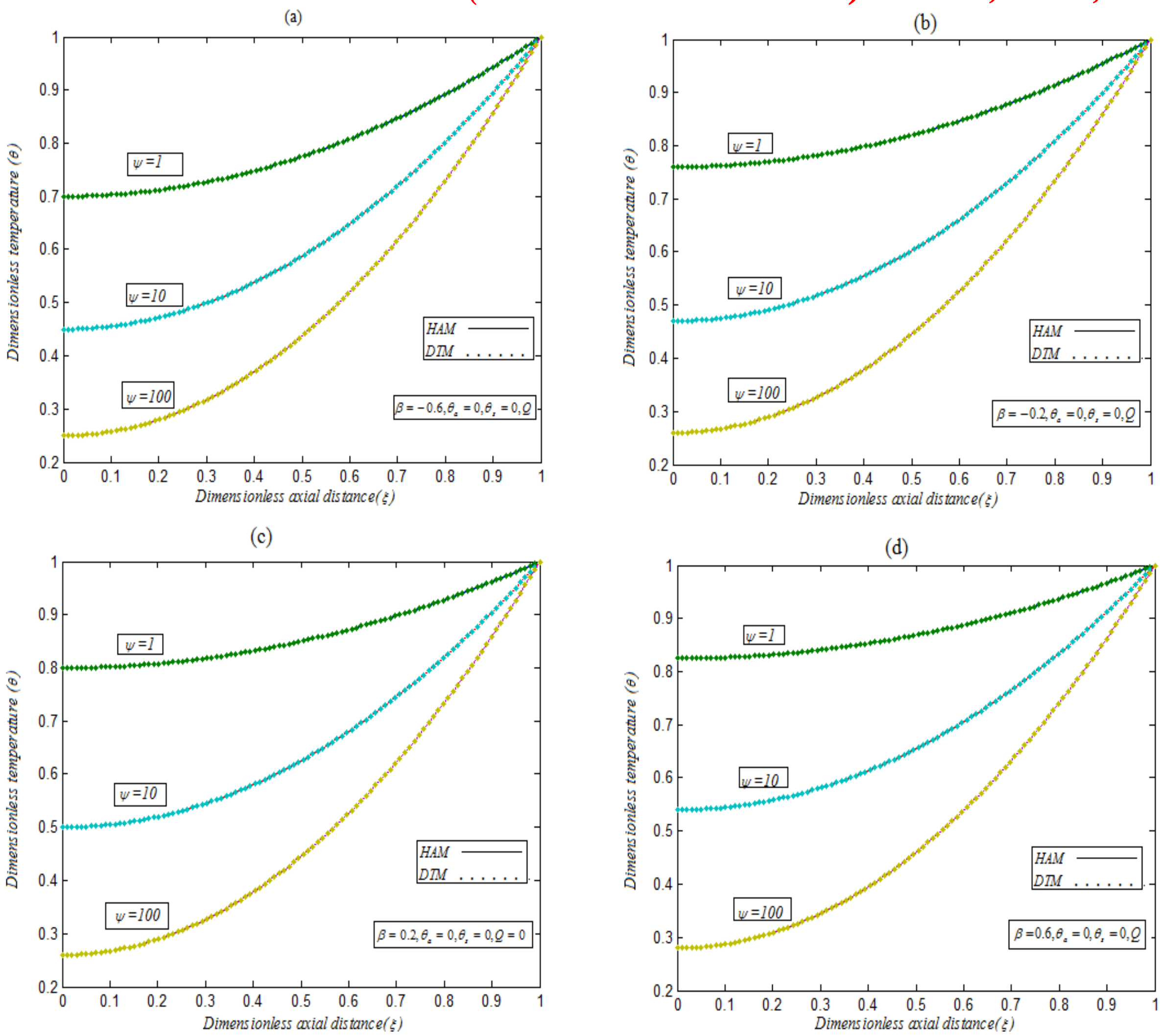

Fig. 2: Dimensionless axial distance ξ versus the dimensionless temperature $\theta(\xi)$. The curves are plotted for various values of the dimensionless parameters ψ, β and some fixed values of θ_a, θ_s, Q using the eqn. (9), when h= - 0.188.

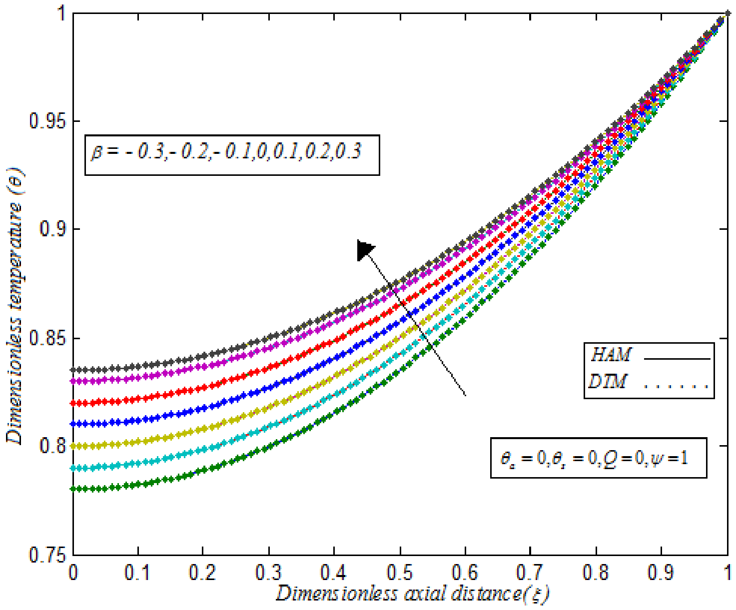

Fig. 3: Dimensionless axial distance ξ versus dimensionless temperature $\theta(\xi)$ for various values of β and some fixed values of other parameter using the eqn. (9), when *h*= - 0.38.

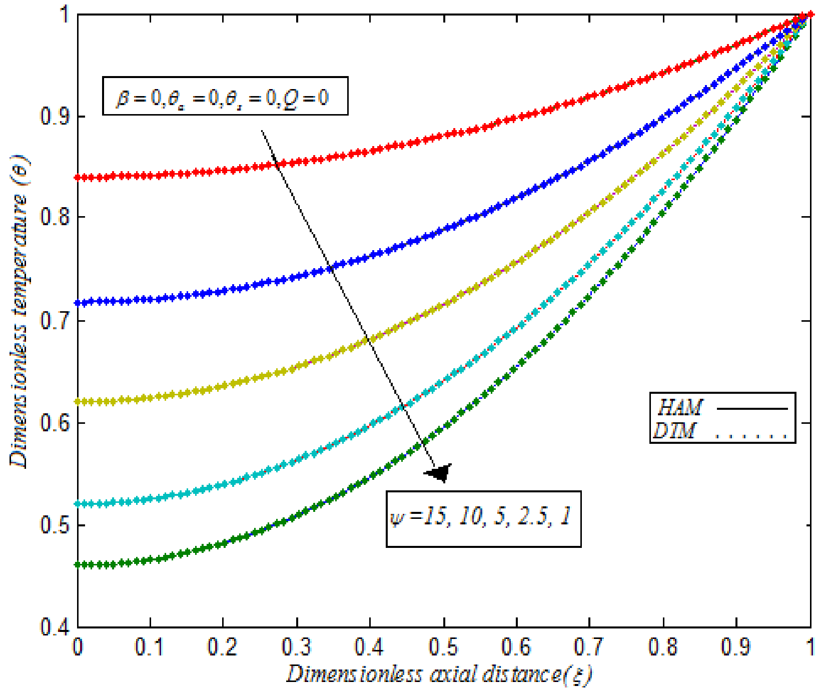

Fig. 4: Dimensionless axial distance ξ versus dimensionless temperature $\theta(\xi)$ for various values of ψ and some fixed values of other parameter using the eqn. (9), when *h*= - 0.173.

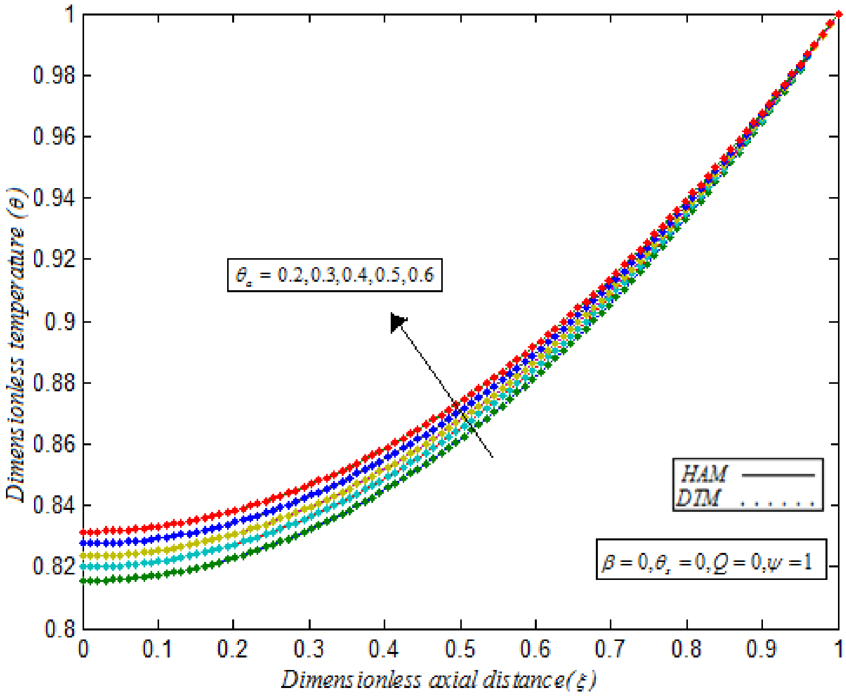

Fig. 5: Dimensionless axial distance ξ versus dimensionless temperature $\theta(\xi)$ for various values of θ_a and some fixed values of other parameter using the eqn. (9), when *h*= - 0.353.

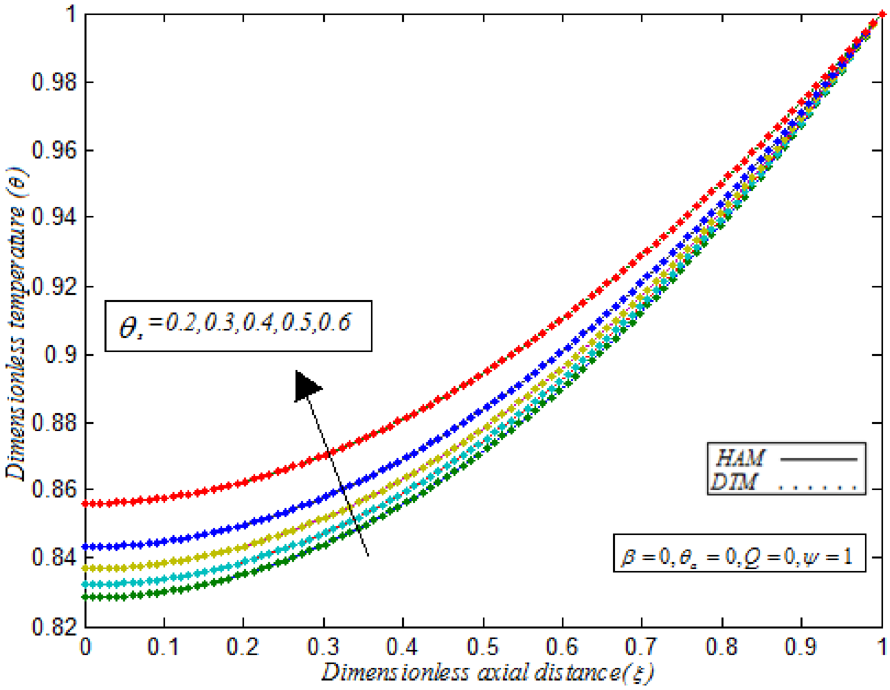

Fig. 6: Dimensionless axial distance ξ versus dimensionless temperature $\theta(\xi)$ for various values of θ_s and some fixed values of other parameter using the eqn. (9), when *h*= - 0.34.

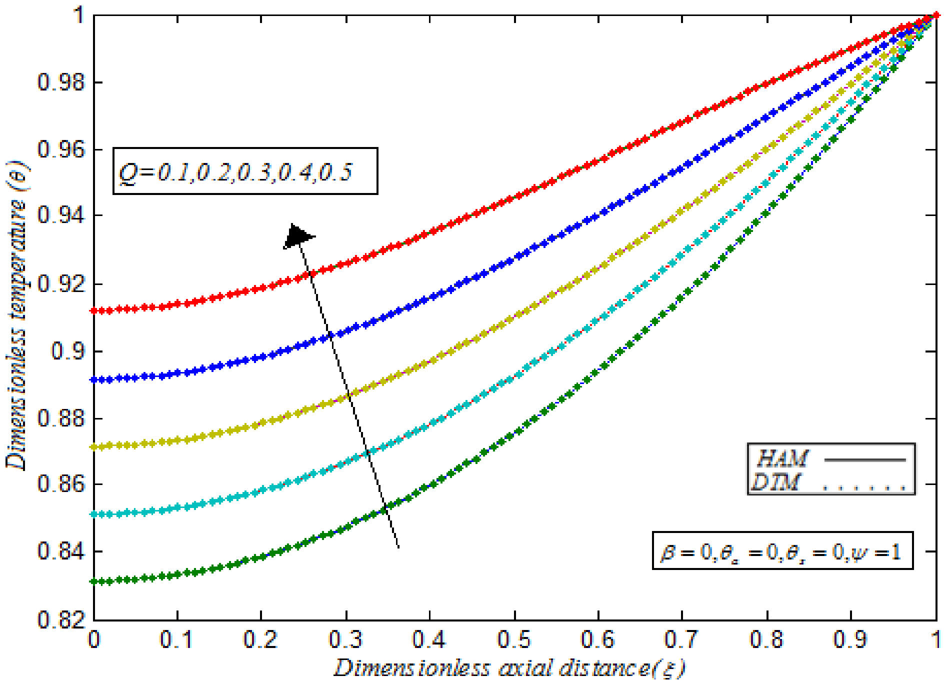

Fig. 7: Dimensionless axial distance ξ versus dimensionless temperature $\theta(\xi)$ for various values of Q and some fixed values of other parameter using the eqn. (9), when *h*= - 0.38.

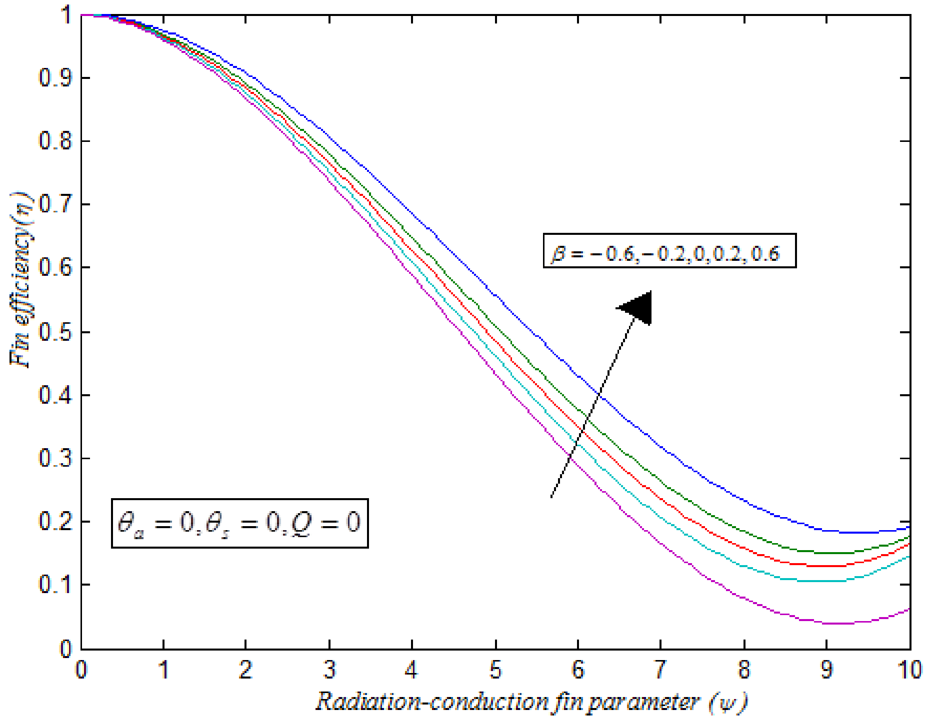

Fig. 8: Fin efficiency (η) versus Radiation-conduction fin parameter (ψ) for various values of β and some fixed values of the other parameter θ_a, θ_s, Q using the eqn. (8),when *h= - 0.34.*

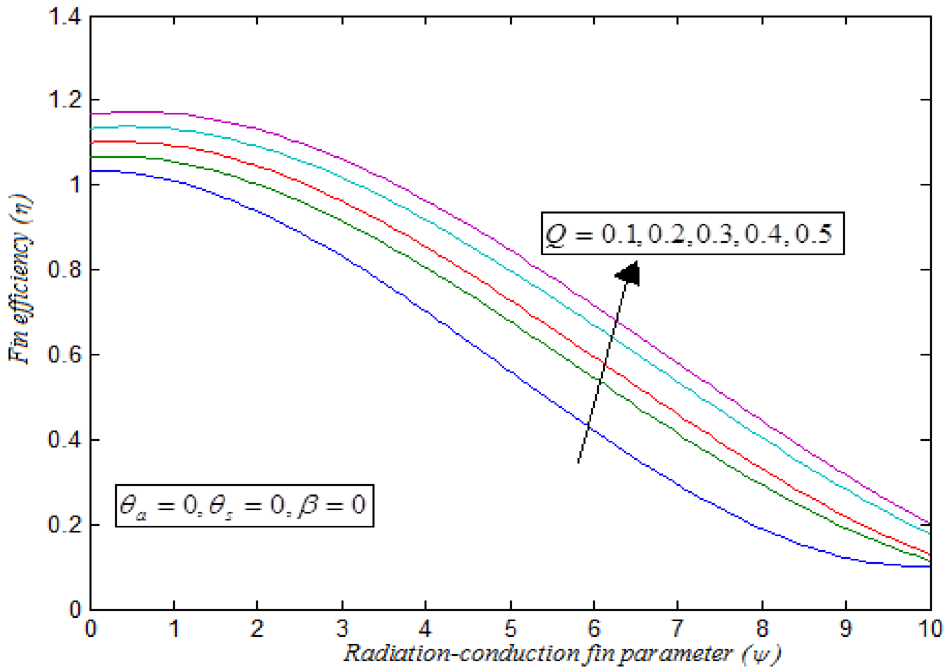

Fig. 9: Fin efficiency (η) versus Radiation-conduction fin parameter (ψ) for various values of Q and some fixed values of the other parameter $\theta_a, \theta_s, \beta$ using the eqn. (8) when $h = -0.2$.

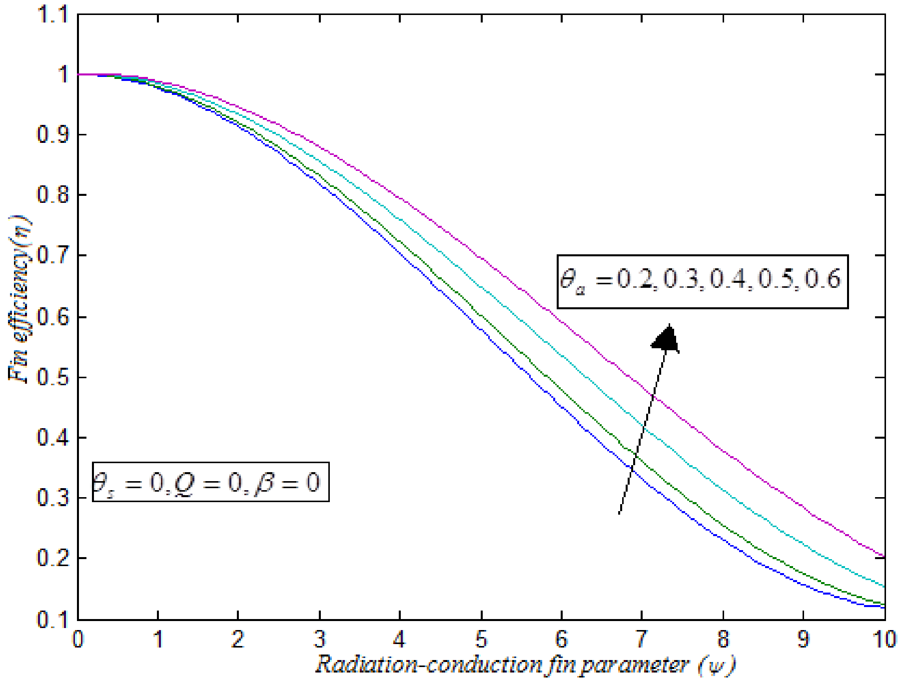

Fig. 10: Fin efficiency (η) versus Radiation-conduction fin parameter (ψ) for various values of θ_a and some fixed values of the other parameter θ_s, β, Q using the eqn. (8) when *h= - 0.28.*

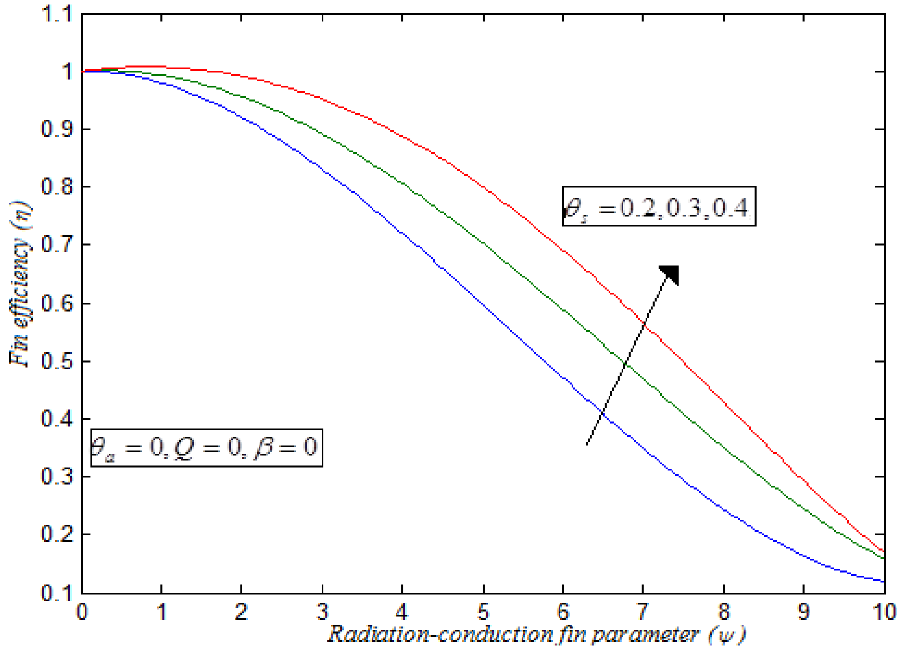

Fig. 11: Fin efficiency (η) versus Radiation-conduction fin parameter (ψ) for various values of θ_s and some fixed values of the other parameter θ_a, β, Q using the eqn. (8) when *h= - 0.24.*

Table 1: Comparison of HAM solution (eqn.(9)) and DTM solution for the dimensionless temperature $\theta(\xi)$

	$\beta = 0.4, \psi = 1$, *h*=0.388		$\beta = 0.2, \psi = 0.5$, *h*=0.541	
ξ	**DTM**	**HAM**	**DTM**	**HAM**
0	0.8133693583	0.8133695000	0.8679116912	0.8679125000

0.05	0.8137822645	0.8137817125	0.8682139335	0.8682140406
0.10	0.8150223528	0.8150234500	0.8691214181	0.8691222475
0.15	0.8170937494	0.8170902000	0.8706363930	0.8706322738
0.20	0.8200033838	0.8200048000	0.8727626384	0.8727628000
0.25	0.8237610702	0.8237640625	0.8755054875	0.8755023438
0.30	0.8283796228	0.8283785500	0.8788718740	0.8788721750
0.35	0.8338750084	0.8338760875	0.8828703906	0.8828708613
0.40	0.8402665394	0.8402656000	0.8875113635	0.8875114000
0.45	0.8475771116	0.8475778125	0.8928069419	0.8928060313
0.50	0.8558334915	0.8558338750	0.8987712062	0.8987706250
0.55	0.8650666613	0.8650686250	0.9054202946	0.9054207438
0.60	0.8753122271	0.8753120000	0.9127725525	0.9127728000
0.65	0.8866109039	0.8866136500	0.9208487049	0.9208464063
0.70	0.8990090866	0.8990092900	0.9296720572	0.9296710000
0.75	0.9125595243	0.9125590625	0.9392687258	0.9392640625
0.80	0.9273221155	0.9273226600	0.9496679059	0.9496630000
0.85	0.9433648484	0.9433653025	0.9609021785	0.9609002500
0.90	0.9607649133	0.9607653800	0.9730078657	0.9730057500
0.95	0.9796100255	0.9796103613	0.9860254404	0.9860209375
1	1.0000000000	1.0000000000	1.0000000000	1.0000000000

6. Conclusion:

In this study, the Homotopy analysis method (HAM) has been applied to solve non-linear differential equation arising in radiative-radial fins with temperature-dependent thermal conductivity problem. The analytical expressions of the dimensionless temperature have been derived by the Homotopy analysis method. The analytical and graphical representations of the fin efficiency are also investigated. Comparison of the results obtained by HAM with those of DTM, showed efficiency of this method to solve strong non-linear equations. In HAM, we can choose h in an appropriate way which controls the convergence of the series. This method can be easily extended to solve the non-linear initial and boundary value problems in physical sciences.

7. Acknowledgement:

Researchers express their gratitude to the Secretary Shri S. Natanagopal, The Madura College Board, Madurai, Dr. K. M. Rajasekaran, The Principal and Dr. S. Muthukumar, Head of the Department of Mathematics, The Madura College (Autonomous), Madurai, Tamil Nadu, India for their constant support and encouragement.

8. References

1. G. Domairry, M.Fazeli,Homotopy analysis method to determine the fin efficiency of convective straight fins with temperature-dependent thermal conductivity, Communications in Nonlinear Science and Numerical stipulation, 14(2009): 519-525.
2. B. Kundu, Analysis of thermal performance and optimization of concentric circular fins under dehumidifying conditions, International Journal of Heat and Mass Transfer, 52(2009): 2646-2659.
3. H. Tari,D.D. Ganji, H. Babazadeh, The application of He'variational iteration method to nonlinear equations arising in heat transfer,Physics Letters A, 362(2007): 213-217.
4. C. Arslanturk, Optimum design of space radiators with temperature-dependent thermal conductivity, Applied thermal Engineering,26(2006): 1149-1157.

5. S. Abbasbandy, The application of Homotopy analysis method to nonlinear equation arising in heat transfer, Physics Letters A, 360(2006): 109-113.
6. F. Khani, A. Aziz, Thermal analysis of a longitudinal trapezoidal fin with temperature-dependent thermal conductivity and heat transfer coefficient, Communications in Nonlinear Science and Numerical Simulation, 15(2010): 590-601.
7. S. J. Liao, The proposed Homotopy analysis technique for the solution of non linear Problems, Ph.D. Thesis, Shanghai Jiao Tong University, 1992.
8. S. J. Liao, An approximate solution technique which does not depend upon small parameters: a special example, Int. J. Non-Linear Mech. 30(1995): 371-380.
9. S. J. Liao, beyond perturbation introduction to the Homotopy analysis method, 1st edn. Chapman and Hall, CRC press, Boca Raton p.336, 2003.
10. S. J. Liao, On the Homotopy analysis method for nonlinear problems, Appl. Math. Comput. 147(2004): 499-513.
11. S. J. Liao, An optimal Homotopy-analysis approach for strongly nonlinear differential equations, Commun. Nonlinear Sci. Numer. Simulat, 15(2010): 2003-2016.
12. S. J. Liao, The Homotopy analysis method in nonlinear differential equations, Springer and Higher education press, 2012.
13. S. J. Liao, An explicit totally analytic approximation of Blasius viscous flow problems. Int J. Nonlinear Mech, 34(1999): 759–78.
14. S. J. Liao, on the analytic solution of magnetohydrodynamic flows non-Newtonian fluid over a stretching sheet, J Fluid Mech, 488(2003): 189–212.
15. S. J. Liao, A new branch of boundary layer flows over a permeable stretching plate, Int J. Nonlinear Mech, 42(2007): 19–30.
16. G. Domairry, H. Bararnia, An approximation of the analytical solution of some nonlinear Heat transfer equations, a survey by using Homotopy analysis method, Adv. Studies Theor. Phys, 2(2008) 507-518.
17. Y. Tan, H. Xu, S.J. Liao, Explicit series solution of travelling waves with a front of Fisher equation, Chaos Solitons Fractals, 31(2007): 462–72.
18. S. Abbasbandy, Soliton solutions for the FitzhughNagumo equation with the Homotopy analysis method, Appl Math Model, 32(2008): 2706–14.
19. J. Cheng, S.J. Liao, R.N. Mohapatra, K. Vajravelu, Series solutions of Nano boundary layer flows by means of the Homotopy analysis method, J Math Anal Appl, 343(2008): 233–245.
20. T. Hayat, Z. Abbas, Heat transfer analysis on MHD flow of a second grade fluid in a channel with porous medium, Chaos Solitons Fractals, 38(2008): 556–567.
21. T. Hayat, R. Nazr, M. Sajid, On the Homotopy solution for Poiseuille flow of a fourth grade fluid, Commun Nonlinear Sci Numer Simul, 15(2010): 581–589.
22. H. Jafari, C. Chun, S. M. Saeidy, Analytical solution for nonlinear gas dynamic Homotopy analysis method, Appl. Math, 4(2009): 149-154.
23. M. Subha, V. Ananthaswamy, and L. Rajendran, Analytical solution of non-linear boundary value problem for the electrohydrodynamic flow equation, International Jouranal of Automation and Control Engineering, 3(2)(2014): 48-56.
24. K. Saravanakumar, V. Ananthaswamy, M. Subha, and L. Rajendran, Analytical Solution of nonlinear boundary valueproblem for in efficiency of convective straight Fins with temperature-dependent thermal conductivity, ISRN Thermodynamics, Article ID 282481(2013): 1-18.

25. V. Ananthaswamy, M. Subha, Analytical expressions for exothermic explosions in aslab, International Journal of Research– Granthaalayah, 1(2)(2014): 22-37.
26. V. Ananthaswamy, S. Uma Maheswari, Analytical expression for the hydrodynamicfluid flow through a porous medium, International Journal of Automation and Control Engineering, 4(2)(2015): 67-76.
27. V. Ananthaswamy, L. Sahaya Amalraj, Thermal stability analysis of reactive hydromagnetic third-grade fluid using Homotopy analysis method, International Journal of Modern Mathematical Sciences, 14 (1)(2016): 25-41.

Appendix: A

Basic Concept of the Homotopy Analysis Method:

Consider the following differential equation:

$$N[u(t)] = 0 \tag{A.1}$$

Where N is a nonlinear operator, t denote an independent variable, $u(t)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao (2012) constructed the so-called zero-order deformation equation as:

$$(1-p)L[\varphi(t;p) - u_0(t)] = phH(t)N[\varphi(t;p)] \tag{A.2}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operator, $u_0(t)$ is an initial guess of $u(t)$, $\varphi(t:p)$ is an unknown function. It is important to note that one has great freedom to choose auxiliary unknowns in HAM. Obviously, when $p = 0$ and $p = 1$, it holds:

$$\varphi(t;0) = u_0(t) \text{ And } \varphi(t;1) = u(t) \tag{A.3}$$

respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_0(t)$ to the solution u (t). Expanding $\varphi(t;p)$ in Taylor series with respect to *p,* we have:

$$\varphi(t;p) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t)p^m \tag{A.4}$$

where

$$u_m(t) = \frac{1}{m!}\frac{\partial^m \varphi(t;p)}{\partial p^m}\Big|_{p=0} \tag{A.5}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter *h,* and the auxiliary function are so properly chosen, the series (A.4) converges at p =1 then we have:

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t). \tag{A.6}$$

Differentiating (A.2) for m times with respect to the embedding parameter p, and then setting p = 0 and finally dividing them by $m!$, we will have the so-called *mth* -order deformation equation as:

$$L[u_m - \chi_m u_{m-1}] = hH(t)\Re_m(\vec{u}_{m-1}) \tag{A.7}$$

where

$$\Re_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\varphi(t;p)]}{\partial p^{m-1}} \tag{A.8}$$

And

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \qquad \text{(A.9)}$$

Applying L^{-1} on both side of equation (A7), we get

$$u_m(t) = \chi_m u_{m-1}(t) + hL^{-1}[H(t)\Re_m(\vec{u}_{m-1})] \qquad \text{(A10)}$$

In this way, it is easily to obtain u_m for $m \ge 1$, at M^{th} order, we have

$$u(t) = \sum_{m=0}^{M} u_m(t) \qquad \text{(A.11)}$$

When $M \to +\infty$, we get an accurate approximation of the original equation (A.1). For the convergence of the above method we refer the reader to Liao [20]. If equation (A.1) admits unique solution, then this method will produce the unique solution.

Appendix B:

Approximate Analytical Expressions of the Non-Linear Differential Eqns. (4)-(6) Using the Homotopy Analysis Method:

$$\frac{d}{d\xi}[(1+\beta(\theta-\theta_a))\frac{d\theta}{d\xi}] - \psi(\theta^4 - \theta_s^4) + Q = 0, \quad 0 \le \xi \le 1 \qquad \text{(B.1)}$$

We construct the Homotopy for the eqn. (B.1) is as follows:

$$(1-p)\left[\frac{d^2\theta}{d\xi^2} + \frac{Q+\psi\theta_s^4}{1-\beta\theta_a}\right] - hp\left[\frac{d^2\theta}{d\xi^2} + \frac{\frac{d^2\theta}{d\xi^2}\beta\theta}{1-\beta\theta_a} - \frac{\psi\theta^4}{1-\beta\theta_a} + \frac{Q+\psi\theta_s^4}{1-\beta\theta_a}\right] = 0 \qquad \text{(B.2)}$$

The approximate solution of the eqn. (B.2) is as follows:

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots \qquad \text{(B.3)}$$

The initial approximations are as follows:

$$\frac{d\theta_0(0)}{d\xi} = 0; \ \theta_0(1) = 1 \qquad \text{(B.4)}$$

$$\frac{d\theta_i(0)}{d\xi} = 0; \ \theta_i(1) = 0, \ i = 1,2,3\ldots.. \qquad \text{(B.5)}$$

Substituting the eqn. (B.3) into the eqn. (B.2) we get

$$\left.\begin{array}{l} (1-p)[\frac{d^2(\theta_0 + p\theta_1 + p^2\theta_2 + \ldots)}{d\xi^2} + \frac{Q+\psi\theta_s^4}{1-\beta\theta_a}] \\ -hp[\frac{d^2\theta}{d\xi^2} + \frac{\frac{d^2(\theta_0 + p\theta_1 + p^2\theta_2 + \ldots)}{d\xi^2}\beta\theta}{1-\beta\theta_a} - \frac{\psi\theta^4}{1-\beta\theta_a} + \frac{Q+\psi\theta_s^4}{1-\beta\theta_a}] \end{array}\right\} = 0 \qquad \text{(B.6)}$$

Comparing the coefficients of like powers p in the eqn. (B.6) we get

$$p^0 : \frac{d^2\theta_0}{d\xi^2} + \frac{Q+\psi\theta_s^4}{1-\beta\theta_a} = 0 \qquad \text{(B.7)}$$

$$p^1 : \frac{d^2\theta_1}{d\xi^2} - h\left[\frac{\frac{d^2\theta_0}{d\xi^2}\beta\theta_0}{1-\beta\theta_a} - \frac{\psi\theta_0^4}{1-\beta\theta_a}\right] = 0 \qquad \text{(B.8)}$$

Solving the eqns. (B.7), (B.8) and using the boundary conditions (B.4) and (B.5) we can obtain the following results:

$$\theta_0 = \frac{s\xi^2}{2u} + 1 - \frac{s}{2u} \qquad \text{(B.9)}$$

$$\theta_1 = \left[\begin{array}{l} \dfrac{1}{2u^2}\left\{\begin{array}{l} -s\beta[s(\dfrac{\xi^4}{4} - \dfrac{\xi^2}{2}) + u\xi^2] + \dfrac{\psi}{8u^3}[s^4(\dfrac{\xi^6}{30} - \dfrac{\xi^5}{5} + \dfrac{\xi^4}{2} - \dfrac{2\xi^3}{3} + \dfrac{\xi^2}{2}) \\ +8s^3u(\dfrac{\xi^5}{20} - \dfrac{\xi^4}{4} + \dfrac{\xi^3}{2} - \dfrac{\xi^2}{2}) + 24s^2u^2(\dfrac{\xi^4}{12} - \dfrac{\xi^3}{3} + \dfrac{\xi^2}{2}) \\ +32su^3(\dfrac{\xi^3}{6} - \dfrac{\xi^2}{2}) + 8u^4\xi^2] \end{array}\right\} \\ -\dfrac{1}{2u^2}\left\{-s\beta[\dfrac{s}{4}(-1+4u)] + \dfrac{\psi}{8u^3}[\dfrac{s^4}{2} - \dfrac{8s^3u}{5} + 6s^2u^2 - \dfrac{32su^3}{3} + 8u^4]\right\} \end{array}\right] \qquad \text{(B.10)}$$

Where,

s and *u* defined in the text eqns. (10) and (11) respectively.

According to the Homotopy analysis method we have

$$\theta = \lim_{p \to 1} \theta(\xi) = \theta_0 + \theta_1 \qquad \text{(B.11)}$$

Using the eqns. (B.9) and (B.10) in (B.11), we obtain the solutions in the text eqns. (9)–(11).

Appendix: C

Nomenclature:

Symbol	Meaning
b	Fin tip length, m
k	Temperature-dependent thermal conductivity, $Wm^{-1}K^{-1}$
k_0	Thermal conductivity at the base temperature, $Wm^{-1}K^{-1}$
q	Volumetric heat generation, $Wm^{=3}$
Q	Dimensionless volumetric heat generation
Q_1	Heat transfer rate from the surfaces of a fin
T	Temperature, K
T_b	Fin's base temperature, K
T_s	Radiation sink temperature, K
ξ	Dimensionless axial distance from the fin
w	Semi-thickness of the fin, m
β	Thermal conductivity parameter
ε	Emissivity
η	Fin efficiency
λ	Slope of the thermal conductivity-temperature curve, K^{-1}
σ	Stefan-Boltzmann constant, $Wm^{-2}K^{-1}$
θ	Dimensionless temperature
θ_s	Dimensionless radiation sink temperature
ψ	Radiation-conduction fin parameter
u, s	Constants