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OPTIMIZATION OF GREEN MANUFACTURING INVENTORY MODEL USING HEXAGONAL FUZZY NUMBER

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Abstract:

Globalization's alarm is ringing in each side of the world, Now-a-days environmental preservation is high on everyone's mind and both of these efforts are steps towards establishing a pollution free society. The primary source of pollutants is the manufacturing sector, which assumes full liability for product processing in order to satisfy a wide variety of customers. They must strike a balance between minimizing the cost, Maximizing the profit, and environmental sustainability in order for the manufacturing sectors to run smoothly, and an inventory model, if properly formulated, would provide the best solution to their problems. In this paper to deal with uncertainty, the Economic Green Manufacturing Quantity (EGMQ) Inventory Model and also fuzzy approach is used. The fuzzy parameters are taken as hexagonal fuzzy numbers. For defuzzification, we using graded mean integration representation method and Extension of lagrangean method is using to find the optimal solution. Finally, numerical example is provided.

Key Words: Defuzzification, Hexagonal Fuzzy Number, Green Manufacturing Model, Extension of Lagrangean Method.

1. Introduction:

Green manufacturing (GM) is quickly gaining traction as a long-term manufacturing solution that has the ability to solve the majority of the world's problems. Sustainable manufacturing operations began to focus on waste reduction in production in the 1980s. Now-a-days, customers are increasingly drawn to natural products. As a result, the market for these items is expanding, prompting the manufacturing industry to employ green manufacturing methods. With the environmental expenses, economic centered manufacturing inventory models become economic centered green manufacturing inventory models.

Melnyk et al. (2002) place a strong emphasis on regulating the flow of environmental trash in order to reduce environmental damage while simultaneously maximizing resource efficiency. According to Seliger et al (2008) the paradigm for sustainable manufacturing shifted from process to product, with an emphasis on resource conservation, energy efficiency, and toxic substance reduction, as well as the creation and use of renewable materials. Maurice, (Maurice,2011), pioneer of formulating environmental oriented inventory models have used deterministic input costs parameters in all his works, but in this work fuzzy input costs parameters are used to handle the problems of uncertainty. Dr. Nivetha Martin and Dr. P. Pandiammal (2016) developed a fuzzy geometric tool is used to find the ideal order quantity in order to meet the manufacturing challenges. In this paper, the demand and cost are taken as hexagonal fuzzy number for fuzzification and Graded mean integration method for defuzzication. In order to fulfil the production challenges, Lagrangean method is used to determine the appropriate order quantity. Finally we conclude with numerical illustration.

2. Definitions and Methodologies:

Fuzzy Set:

A fuzzy set $A^{\emptyset}$ in a universe of discourse X is defined as $A^{\emptyset} = \{(x, \mu_{A^{\emptyset}}(x) : x \epsilon X\}$. Here $\mu_{A^{\emptyset}} : X \rightarrow [0,1]$is a mapping called the membership value of $x \in X$ in a fuzzy set $A^{\emptyset}$.

Hexagonal Fuzzy Numbers:

A fuzzy number $A^{\emptyset}$ is a hexagonal fuzzy number denoted by$A^{\emptyset} = (a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ Where $(a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ are real numbers satisfying $(a_2 - a_1 \le a_3 - a_2)$ and $(a_5 - a_4 \le a_6 - a_5)$ and its membership function $\mu_{A^{\emptyset}}(x)$is given as

$$\mu_{\tilde{A}}(x)=\begin{cases}0, x<a_1\\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), a_1\le x\le a_2\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), a_2\le x\le a_3\\ 1, a_3\le x\le a_4\\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), a_4\le x\le a_5\\ 0, x>a_6\end{cases}$$

Graded Mean Integration Representation Method:

If $A^{\emptyset}=(g,\ h,\ i,\ j,\ k,\ l\ )$is a hexagonal fuzzy number then the graded mean representation (GMIR) method of $A^{\emptyset}$ is defined as

$$P(A^{\emptyset})=\frac{1}{12}(g+3h+2i+2j+3k+l)$$

The Function Principle:

Suppose $\widetilde{K}=\left(k_{1,}k_{2,}k_{3,}k_4,k_{5,}k_6\right)$ *and*
$\tilde{L}=\left(l_{1,}l_{2,}l_{3,}l_4,l_{5,}l_6\right)$are two hexagonal fuzzy numbers then,
The addition of $\widetilde{K}$ and $\tilde{L}$ is

$$\widetilde{K}\oplus\tilde{L}=\left(k_1+l_{1,}k_2+l_{2,}k_3+l_{3,}k_4+l_4,k_5l_5,k_6+l_6\right)$$

The multiplication of $\widetilde{K}$ and $\tilde{L}$ is

$$\widetilde{K}\otimes\tilde{L}=\left(k_1l_{1,}k_2l_{2,}k_3l_{3,}k_4l_4,k_5l_5,k_6l_6\right)$$

The subtraction of $\widetilde{K}$and $\tilde{L}$ is

$$\widetilde{K}\ominus\tilde{L}=\begin{pmatrix}k_1-l_6,k_2-l_{5,}k_3-l_{4,}k_4-l_{3,}k_5-l_{2,}\\ k_6-l_{1,}\end{pmatrix}$$

The division of $\widetilde{K}$and $\tilde{L}$ is

$$\widetilde{K}\oslash\tilde{L}=\left(\frac{k_1}{l_6},\frac{k_2}{l_5},\frac{k_3}{l_4},\frac{k_4}{l_3},\frac{k_5}{l_2},\frac{k_6}{l_1}\right)$$

Let $\alpha\epsilon$R, Then

$$\alpha\ge 0,\ \alpha\otimes\widetilde{K}=(\alpha k_1,\alpha k_2,\alpha k_3,\alpha k_4,\alpha k_5,\alpha k_6)$$
$$\alpha<0,\ \alpha\otimes\tilde{L}=(\alpha l_6,\alpha l_5,\alpha l_4,\alpha l_3,\ \alpha l_2,\alpha l_1)$$

Lagrangean Method:

By Minimizing, c = f (b)

Subject to $h_i(b)\ge 0$, i = 1,2,……,m.

The non-negativity constraints b ≥ 0, if any, are covered in the n constraints. Then, the procedure of Extension of the Lagrangean method involves the below steps.

Step 1: Solve the unconstrained problem,

Minimize c = f (b).

If the resultant optimum satisfies all the constraints, end the step because all constraints are excessive. Otherwise, set k = 1 and move on to step 2.

Step 2: Activate any k constraints and optimize f(b) subject to the k active constraints by the Lagrangean method. If the resultant solution is feasible with respect to the enduring constraints, stop the process because it is a local optimum. Or else, activate another set of k - constraints and renew the step. If all remaining sets of active constraints taken k at a time are considered without confronting a feasible solution, move on to step 3.

Step 3: If k = n, stop; no feasible solution exists. Or else, set k = k+1 and go to step 2. By using the above Lagrangean method, we discuss the fuzzy inventory model by changing the crisp quantity into fuzzy quantity. As a result, we can get optimal solution.

3. Notations:

D - Demand (units/ time),
S– Inventory carrying cost,
W– Setup cost (dollar/batch),
G_c – Green cost per unit,
E_s – Environmental sustainability cost/unit,
M_c – Manufacturing cost / unit (dollar/unit),
q*-Order quantity,
T_c-Total cost,

$\tilde{S}$ – Fuzzy Inventory carrying cost,
$\widetilde{W}$ –Fuzzy Setup cost (dollar/batch),
$\widetilde{G_c}$ – Fuzzy Green cost per unit,
$\widetilde{E_s}$– Fuzzy Environmental sustainability cost/unit,
$\widetilde{M_c}$– Fuzzy Manufacturing cost / unit (dollar/unit),
$T_{\tilde{C}}$- Fuzzy Total cost,
$\widetilde{D}$ – Fuzzy Demand,
Q*-Optimal order

4. Crisp Sense:

First, we consider a Green manufacturing inventory model in crisp sense; here the total inventory cost is obtained by the equation.

$$T_c = \frac{WD}{q} + M_c(D) + \frac{Sq}{2} + \frac{G_cD}{q} + \frac{E_sD}{q}$$

Differentiate partially with respect to q and equate it to zero,

$$\frac{\partial T_c}{\partial q} = 0$$

$$\Rightarrow -\frac{WD}{q^2} + \frac{S}{2} - \frac{G_cD}{q^2} - \frac{E_sD}{q^2}$$

$$\Rightarrow q = \sqrt{\frac{2(W+G_c+E_s)}{S}}$$

5. Fuzzy Sense:

$$\widetilde{G_C} = (g_{c_1}, g_{c_2}, g_{c_3}, g_{c_4}, g_{c_5}, g_{c_6});$$
$$\widetilde{E_s} = (e_{s_1}, e_{s_2}, e_{s_3}, e_{s_4}, e_{s_5}, e_{s_6});$$
$$\widetilde{D} = (d_1, d_2, d_3, d_4, d_5, d_6)$$
$$\widetilde{W} = (w_1, w_2, w_3, w_4, w_5, w_6);$$
$$\tilde{S} = (s_1, s_2, s_3, s_4, s_5, s_6)$$

$$T_{\tilde{C}} = \begin{bmatrix} \frac{w_1d_1}{q} + m_{c_1}d_1 + \frac{s_1q}{2} + \frac{g_{c_1}d_1}{q} + \frac{e_{s_1}d_1}{q}, \\ \frac{w_2d_2}{q} + m_{c_2}d_2 + \frac{s_2q}{2} + \frac{g_{c_2}d_2}{q} + \frac{e_{s_2}d_2}{q}, \\ \frac{w_3d_3}{q} + m_{c_3}d_3 + \frac{s_3q}{2} + \frac{g_{c_3}d_3}{q} + \frac{e_{s_3}d_3}{q}, \\ \frac{w_4d_4}{q} + m_{c_4}d_4 + \frac{s_4q}{2} + \frac{g_{c_4}d_4}{q} + \frac{e_{s_4}d_4}{q}, \\ \frac{w_5d_5}{q} + m_{c_5}d_5 + \frac{s_5q}{2} + \frac{g_{c_5}d_5}{q} + \frac{e_{s_5}d_5}{q}, \\ \frac{w_6d_6}{q} + m_{c_6}d_6 + \frac{s_6q}{2} + \frac{g_{c_6}d_6}{q} + \frac{e_{s_6}d_6}{q} \end{bmatrix}$$

By Graded mean integration representation method,

$$F(T_{\tilde{C}}) = 0$$

$$\Rightarrow \frac{1}{12}\begin{bmatrix} \left(\frac{w_1d_1}{q} + m_{c_1}d_1 + \frac{s_1q}{2} + \frac{g_{c_1}d_1}{q} + \frac{e_{s_1}d_1}{q}\right) + \\ 3\left(\frac{w_2d_2}{q} + m_{c_2}d_2 + \frac{s_2q}{2} + \frac{g_{c_2}d_2}{q} + \frac{e_{s_2}d_2}{q}\right) + \\ 2\left(\frac{w_3d_3}{q} + m_{c_3}d_3 + \frac{s_3q}{2} + \frac{g_{c_3}d_3}{q} + \frac{e_{s_3}d_3}{q}\right) + \\ 2\left(\frac{w_4d_4}{q} + m_{c_4}d_4 + \frac{s_4q}{2} + \frac{g_{c_4}d_4}{q} + \frac{e_{s_4}d_4}{q}\right) + \\ 3\left(\frac{w_5d_5}{q} + m_{c_5}d_5 + \frac{s_5q}{2} + \frac{g_{c_5}d_5}{q} + \frac{e_{s_5}d_5}{q}\right) + \\ \left(\frac{w_6d_6}{q} + m_{c_6}d_6 + \frac{s_6q}{2} + \frac{g_{c_6}d_6}{q} + \frac{e_{s_6}d_6}{q}\right) \end{bmatrix}$$

Differentiate partially with respect to q and equate it to zero

$$\frac{\partial F(T_{\tilde{C}})}{\partial q} = 0$$

$$\Rightarrow \frac{1}{12}\begin{bmatrix} \left(\frac{s_1}{2}-\frac{w_1 d_1}{q^2}-\frac{g_{c_1} d_1}{q^2}-\frac{e_{s_1} d_1}{q^2}\right)+ \\ 3\left(\frac{s_2}{2}-\frac{w_2 d_2}{q^2}-\frac{g_{c_2} d_2}{q^2}+\frac{e_{s_2} d_2}{q^2}\right)+ \\ 2\left(\frac{s_3}{2}-\frac{w_3 d_3}{q^2}-\frac{g_{c_3} d_3}{q^2}-\frac{e_{s_3} d_3}{q^2}\right)+ \\ 2\left(\frac{s_4}{2}-\frac{w_4 d_4}{q^2}-\frac{g_{c_4} d_4}{q^2}-\frac{e_{s_4} d_4}{q^2}\right)+ \\ 3\left(\frac{s_5}{2}-\frac{w_5 d_5}{q^2}+\frac{g_{c_5} d_5}{q^2}+\frac{e_{s_5} d_5}{q^2}\right)+ \\ \left(\frac{s_6}{2}-\frac{w_6 d_6}{q^2}-\frac{g_{c_6} d_6}{q^2}-\frac{e_{s_6} d_6}{q^2}\right) \end{bmatrix}=0$$

$$q^*=\sqrt{\frac{2\begin{bmatrix}(w_1d_1+g_{c_1}d_1+e_{s_1}d_1)+3(w_2d_2+g_{c_2}d_2+e_{s_2}d_2)+ \\ 2(w_3d_3+g_{c_3}d_3+e_{s_3}d_3)+2(w_4d_4+g_{c_4}d_4+e_{s_4}d_4) \\ +3(w_5d_5+g_{c_5}d_5+e_{s_5}d_5)+(w_6d_6+g_{c_6}d_6+e_{s_6}d_6)\end{bmatrix}}{(s_1+3s_2+2s_3+2s_4+3s_5+s_6)}}$$

5. Optimal Solution Obtained by Lagrangean Method:

Suppose the fuzzy order quantity

$$\tilde{q} = (q_1, q_2, q_3, q_4, q_5, q_6)$$

By arithmetic principle and applying hexagonal fuzzy numbers

$$T_{\tilde{C}} = \begin{bmatrix} \frac{w_1 d_1}{q_6,}+m_{c_1} d_1+\frac{s_1 q_1,}{2}+\frac{g_{c_1} d_1}{q_6,}+\frac{e_{s_1} d_1}{q_6,}, \\ \frac{w_2 d_2}{q_5}+m_{c_2} d_2+\frac{s_2 q_2}{2}+\frac{g_{c_2} d_2}{q_5}+\frac{e_{s_2} d_2}{q_5}, \\ \frac{w_3 d_3}{q_4}+m_{c_3} d_3+\frac{s_3 q_3}{2}+\frac{g_{c_3} d_3}{q_4}+\frac{e_{s_3} d_3}{q_4}, \\ \frac{w_4 d_4}{q_3}+m_{c_4} d_4+\frac{s_4 q_4}{2}+\frac{g_{c_4} d_4}{q_3}+\frac{e_{s_4} d_4}{q_3}, \\ \frac{w_5 d_5}{q_2}+m_{c_5} d_5+\frac{s_5 q_5}{2}+\frac{g_{c_5} d_5}{q_2}+\frac{e_{s_5} d_5}{q_2}, \\ \frac{w_6 d_6}{q_1}+m_{c_6} d_6+\frac{s_6 q_6}{2}+\frac{g_{c_6} d_6}{q_1}+\frac{e_{s_6} d_6}{q_1} \end{bmatrix}$$

By using graded mean integration representation method

$$F(T_{\tilde{c}}) = \frac{1}{12}\begin{bmatrix} \left(\frac{w_1 d_1}{q_6,}+m_{c_1} d_1+\frac{s_1 q_1,}{2}+\frac{g_{c_1} d_1}{q_6,}+\frac{e_{s_1} d_1}{q_6,}\right)+ \\ 3\left(\frac{w_2 d_2}{q_5}+m_{c_2} d_2+\frac{s_2 q_2}{2}+\frac{g_{c_2} d_2}{q_5}+\frac{e_{s_2} d_2}{q_5}\right)+ \\ 2\left(\frac{w_3 d_3}{q_4}+m_{c_3} d_3+\frac{s_3 q_3}{2}+\frac{g_{c_3} d_3}{q_4}+\frac{e_{s_3} d_3}{q_4}\right)+ \\ 2\left(\frac{w_4 d_4}{q_3}+m_{c_4} d_4+\frac{s_4 q_4}{2}+\frac{g_{c_4} d_4}{q_3}+\frac{e_{s_4} d_4}{q_3}\right)+ \\ 3\left(\frac{w_5 d_5}{q_2}+m_{c_5} d_5+\frac{s_5 q_5}{2}+\frac{g_{c_5} d_5}{q_2}+\frac{e_{s_5} d_5}{q_2}\right)+ \\ \left(\frac{w_6 d_6}{q_1}+m_{c_6} d_6+\frac{s_6 q_6}{2}+\frac{g_{c_6} d_6}{q_1}+\frac{e_{s_6} d_6}{q_1}\right) \end{bmatrix} \qquad (1)$$

With $0 < q_1 \le q_2 \le q_3 \le q_4 \le q_5 \le q_6$

It can be written as

$$q_2 - q_1 \ge 0,$$
$$q_3 - q_2 \ge 0,$$
$$q_4 - q_3 \ge 0,$$
$$q_5 - q_4 \ge 0,$$
$$q_6 - q_5 \ge 0,$$
$$q_1 > 0.$$

Step 1:

Differentiate (1) partially with respect to $q_1, q_2, q_3, q_4, q_5, q_6$ and equate them to zero.

$$\frac{\partial}{\partial q_1}(F(T_{\tilde{c}})) = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2}-\frac{w_6 d_6}{{q_1}^2}-\frac{g_{c_6} d_6}{{q_1}^2}-\frac{e_{s_6} d_6}{{q_1}^2}\right]=0$$

$$\Rightarrow \frac{s_1}{2} = \frac{w_6 d_6}{{q_1}^2} + \frac{g_{c_6} d_6}{{q_1}^2} + \frac{e_{s_6} d_6}{{q_1}^2}$$

$$\Rightarrow q_1 = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6)]}{(s_1)}}$$

$$\frac{\partial}{\partial q_2}(\mathrm{F}(T_{\tilde{c}})) = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_5}{2} - \frac{w_5 d_5}{{q_2}^2} - \frac{g_{c_5} d_5}{{q_2}^2} - \frac{e_{s_5} d_5}{{q_2}^2}\right] = 0$$

$$\Rightarrow \frac{s_5}{2} = \frac{w_5 d_5}{{q_2}^2} + \frac{g_{c_5} d_5}{{q_2}^2} + \frac{e_{s_5} d_5}{{q_2}^2}$$

$$\Rightarrow q_2 = \sqrt{\frac{2[(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5)]}{(s_2)}}$$

$$\frac{\partial}{\partial q_3}\left(\mathrm{F}(T_{\tilde{c}})\right) = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_4}{2} - \frac{w_4 d_4}{{q_3}^2} - \frac{g_{c_4} d_4}{{q_3}^2} - \frac{e_{s_4} d_4}{{q_3}^2}\right] = 0$$

$$\Rightarrow \frac{s_4}{2} = \frac{w_4 d_4}{{q_3}^2} + \frac{g_{c_4} d_4}{{q_3}^2} + \frac{e_{s_4} d_4}{{q_3}^2}$$

$$\Rightarrow q_3 = \sqrt{\frac{2[(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4)]}{(s_3)}}$$

$$\frac{\partial}{\partial q_4}(\mathrm{F}(T_{\tilde{c}})) = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_3}{2} - \frac{w_3 d_3}{{q_4}^2} - \frac{g_{c_3} d_3}{{q_4}^2} - \frac{e_{s_3} d_3}{{q_4}^2}\right] = 0$$

$$\Rightarrow \frac{s_3}{2} = \frac{w_3 d_3}{{q_4}^2} + \frac{g_{c_3} d_3}{{q_4}^2} + \frac{e_{s_3} d_3}{{q_4}^2}$$

$$\Rightarrow q_4 = \sqrt{\frac{2[(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3)]}{(s_4)}}$$

$$\frac{\partial}{\partial q_5}(\mathrm{F}(T_{\tilde{c}})) = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_2}{2} - \frac{w_2 d_2}{{q_5}^2} - \frac{g_{c_2} d_2}{{q_5}^2} - \frac{e_{s_2} d_2}{{q_5}^2}\right] = 0$$

$$\Rightarrow \frac{s_2}{2} = \frac{w_2 d_2}{{q_5}^2} + \frac{g_{c_2} d_2}{{q_5}^2} + \frac{e_{s_2} d_2}{{q_5}^2}$$

$$\Rightarrow q_5 = \sqrt{\frac{2[(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2)]}{(s_5)}}$$

$$\frac{\partial}{\partial q_6}(\mathrm{F}(T_{\tilde{c}})) = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_1 d_1}{{q_6}^2} - \frac{g_{c_1} d_1}{{q_6}^2} - \frac{e_{s_1} d_1}{{q_6}^2}\right] = 0$$

$$\Rightarrow \frac{s_1}{2} = \frac{w_1 d_1}{{q_6}^2} + \frac{g_{c_1} d_1}{{q_6}^2} + \frac{e_{s_1} d_1}{{q_6}^2}$$

$$\Rightarrow q_6 = \sqrt{\frac{2[(w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{(s_6)}}$$

It does not satisfy the conditions

$$0 < q_1 \le q_2 \le q_3 \le q_4 \le q_5 \le q_6$$

Step 2:

Set k = 1(fix the constraints as 1)

Convert the inequality constraints $q_2 - q_1 \ge 0$ into an equality constraints $q_2 - q_1 = 0$.

Minimize F ($T_{\tilde{c}}$) with respect to $q_2 - q_1 = 0$ by the lagrangean method.

L $(q_1, q_2, q_3, q_4, q_5, q_6, \lambda)$ = F $(T_{\tilde{c}})$ –λ $(q_2 - q_1)$

$$= \frac{1}{12}\left[\begin{array}{l} \left(\frac{w_1 d_1}{q_{6,}} + m_{c_1} d_1 + \frac{s_1 q_{1,}}{2} + \frac{g_{c_1} d_1}{q_{6,}} + \frac{e_{s_1} d_1}{q_{6,}}\right) + \\ 3\left(\frac{w_2 d_2}{q_5} + m_{c_2} d_2 + \frac{s_2 q_2}{2} + \frac{g_{c_2} d_2}{q_5} + \frac{e_{s_2} d_2}{q_5}\right) + \\ 2\left(\frac{w_3 d_3}{q_4} + m_{c_3} d_3 + \frac{s_3 q_3}{2} + \frac{g_{c_3} d_3}{q_4} + \frac{e_{s_3} d_3}{q_4}\right) + \\ 2\left(\frac{w_4 d_4}{q_3} + m_{c_4} d_4 + \frac{s_4 q_4}{2} + \frac{g_{c_4} d_4}{q_3} + \frac{e_{s_4} d_4}{q_3}\right) + \\ 3\left(\frac{w_5 d_5}{q_2} + m_{c_5} d_5 + \frac{s_5 q_5}{2} + \frac{g_{c_5} d_5}{q_2} + \frac{e_{s_5} d_5}{q_2}\right) + \\ \left(\frac{w_6 d_6}{q_1} + m_{c_6} d_6 + \frac{s_6 q_6}{2} + \frac{g_{c_6} d_6}{q_1} + \frac{e_{s_6} d_6}{q_1}\right) \end{array}\right]$$

$$- \lambda (q_2 - q_1) = 0 \quad (2)$$

Differentiate (2) partially with respect to $q_1, q_2, q_3, q_4, q_5, q_6$, λ and equate them to zero.

$$\frac{\partial L}{\partial q_1} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_6 d_6}{{q_1}^2} - \frac{g_{c6} d_6}{{q_1}^2} - \frac{e_{s6} d_6}{{q_1}^2}\right] + \lambda = 0 \qquad (3)$$

$$\frac{\partial L}{\partial q_2} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_5}{2} - \frac{w_5 d_5}{{q_2}^2} + \frac{g_{c5} d_5}{{q_2}^2} + \frac{e_{s5} d_5}{{q_2}^2}\right] - \lambda = 0 \qquad (4)$$

$$\frac{\partial L}{\partial q_3} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_4}{2} - \frac{w_4 d_4}{{q_3}^2} - \frac{g_{c_4} d_4}{{q_3}^2} - \frac{e_{s_4} d_4}{{q_3}^2}\right] = 0 \qquad (5)$$

$$\frac{\partial L}{\partial q_4} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_3}{2} - \frac{w_3 d_3}{{q_4}^2} - \frac{g_{c_3} d_3}{{q_4}^2} - \frac{e_{s_3} d_3}{{q_4}^2}\right] = 0$$

$$\frac{s_3}{2} = \frac{w_3 d_3}{{q_4}^2} + \frac{g_{c_3} d_3}{{q_4}^2} + \frac{e_{s_3} d_3}{{q_4}^2}$$

$$q_4 = \sqrt{\frac{2[(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3)]}{(s_4)}} \qquad (6)$$

$$\frac{\partial L}{\partial q_5} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_2}{2} - \frac{w_2 d_2}{{q_5}^2} - \frac{g_{c2} d_2}{{q_5}^2} - \frac{e_{s_2} d_2}{{q_5}^2}\right] = 0$$

$$\frac{s_2}{2} = \frac{w_2 d_2}{{q_5}^2} + \frac{g_{c_2} d_2}{{q_5}^2} + \frac{e_{s_2} d_2}{{q_5}^2}$$

$$q_5 = \sqrt{\frac{2[(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2)]}{(s_5)}}, \qquad (7)$$

$$\frac{\partial L}{\partial q_6} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_1 d_1}{{q_6}^2} - \frac{g_{c_1} d_1}{{q_6}^2} - \frac{e_{s_1} d_1}{{q_6}^2}\right] = 0$$

$$\frac{s_1}{2} = \frac{w_1 d_1}{{q_6}^2} + \frac{g_{c_1} d_1}{{q_6}^2} + \frac{e_{s_1} d_1}{{q_6}^2}$$

$$q_6 = \sqrt{\frac{2[(w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{(s_6)}} \qquad (8)$$

$$\frac{\partial L}{\partial \lambda} = 0$$

$$\Rightarrow -(q_2 - q_1) = 0 \qquad (9)$$

(3) + (4) and from the equation (9) we obtain,

$$q_1 = q_2 = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5)]}{s_1 + 3s_2}}$$

$$q_3 = \sqrt{\frac{2[(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4)]}{(s_3)}},$$

$$q_4 = \sqrt{\frac{2[(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3)]}{(s_4)}}$$

$$q_5 = \sqrt{\frac{2[(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2)]}{(s_5)}},$$

$$q_6 = \sqrt{\frac{2[(w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{(s_6)}}$$

$$q_1 = q_2 > q_3 > q_4 > q_5 > q_6.$$

It does not satisfy the local optimum

Step 3:

Set k = 2(fix the constraints as 2)

Convert the inequality constraints $q_2 - q_1 \geq 0$ and $q_3 - q_2 \geq 0$ into an equality constraints $q_3 - q_2 = 0 and\ q_2 - q_1 = 0$.

Minimize F $((T_{\tilde{c}})$ with respect to $q_3 - q_2 = 0 and$ $q_2 - q_1 = 0$ by the lagrangean method.

$$L(q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2) = F(T_{\tilde{c}}) - \lambda_1 (q_2 - q_1) - \lambda_2 (q_3 - q_2) \quad (10)$$

Differentiate (10) partially with respect to $q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2$ and equate them to zero.

$$\frac{\partial L}{\partial q_1} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_6 d_6}{q_1^2} - \frac{g_{c_6} d_6}{q_1^2} - \frac{e_{s_6} d_6}{q_1^2}\right] + \lambda_1 = 0 \quad (11)$$

$$\frac{\partial L}{\partial q_2} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_5}{2} - \frac{w_5 d_5}{q_2^2} + \frac{g_{c_5} d_5}{q_2^2} + \frac{e_{s_5} d_5}{q_2^2}\right] - \lambda_1 + \lambda_2 = 0 \quad (12)$$

$$\frac{\partial L}{\partial q_3} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_4}{2} - \frac{w_4 d_4}{q_3^2} - \frac{g_{c_4} d_4}{q_3^2} - \frac{e_{s_4} d_4}{q_3^2}\right] - \lambda_2 = 0 \quad (13)$$

$$\frac{\partial L}{\partial q_4} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_3}{2} - \frac{w_3 d_3}{q_4^2} - \frac{g_{c_3} d_3}{q_4^2} - \frac{e_{s_3} d_3}{q_4^2}\right] = 0 \quad (14)$$

$$\frac{\partial L}{\partial q_5} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_2}{2} - \frac{w_2 d_2}{q_5^2} - \frac{g_{c_2} d_2}{q_5^2} + \frac{e_{s_2} d_2}{q_5^2}\right] = 0 \quad (15)$$

$$\frac{\partial L}{\partial q_6} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_1 d_1}{q_6^2} - \frac{g_{c_1} d_1}{q_6^2} - \frac{e_{s_1} d_1}{q_6^2}\right] = 0 \quad (16)$$

$$\frac{\partial L}{\partial \lambda_1} = 0$$

$$\Rightarrow -(q_2 - q_1) = 0 \quad \text{and}$$

$$\frac{\partial L}{\partial \lambda_2} = 0$$

$$\Rightarrow -(q_4 - q_3) = 0 \quad (17)$$

(11) + (12)+(13) and From the equation (17) we get,

$$q_1 = q_2 = q_3 = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5) + 2(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4)]}{s_1 + 3s_2 + 2s_3}}$$

$$q_4 = \sqrt{\frac{2[(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3)]}{(s_4)}}$$

$$q_5 = \sqrt{\frac{2[(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2)]}{(s_5)}},$$

$$q_6 = \sqrt{\frac{2[(w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{(s_6)}}$$

$$q_1 = q_2 = q_3 > q_4 > q_5 > q_6.$$

It does not satisfy the local optimum

Step 4:

Set k = 3(fix the constraints as 3)

Convert the inequality constraints $q_2 - q_1 \geq 0$, $q_3 - q_2 \geq 0$ and $q_4 - q_3 \geq 0$ into an equality constraints $q_3 - q_2 = 0, q_2 - q_1 = 0$ *and* $q_4 - q_3 = 0.$

Minimize F $((T_{\tilde{c}})$ with respect to $q_3 - q_2 = 0,$ $q_2 - q_1 = 0$ *and* $q_4 - q_3 = 0$ by the lagrangean method.

$$L(q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2, \lambda_3) = F(T_{\tilde{c}}) - \lambda_1(q_2 - q_1) - \lambda_2(q_3 - q_2) - \lambda_3(q_4 - q_3) \quad (18)$$

Differentiate (18) partially with respect to $q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2, \lambda_3$ and equate them to zero

$$\frac{\partial L}{\partial q_1} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_6 d_6}{q_1^2} - \frac{g_{c_6} d_6}{q_1^2} - \frac{e_{s_6} d_6}{q_1^2}\right] + \lambda_1 = 0 \quad (19)$$

$$\frac{\partial L}{\partial q_2} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_5}{2} - \frac{w_5 d_5}{q_2^2} + \frac{g_{c_5} d_5}{q_2^2} + \frac{e_{s_5} d_5}{q_2^2}\right] - \lambda_1 + \lambda_2 = 0 \quad (20)$$

$$\frac{\partial L}{\partial q_3} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_4}{2} - \frac{w_4 d_4}{q_3^2} - \frac{g_{c_4} d_4}{q_3^2} - \frac{e_{s_4} d_4}{q_3^2}\right] - \lambda_2 + \lambda_3 = 0 \quad (21)$$

$$\frac{\partial L}{\partial q_4} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_3}{2} - \frac{w_3 d_3}{q_4^2} - \frac{g_{c_3} d_3}{q_4^2} - \frac{e_{s_3} d_3}{q_4^2}\right] - \lambda_3 = 0 \qquad (22)$$

$$\frac{\partial L}{\partial q_5} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_2}{2} - \frac{w_2 d_2}{q_5^2} - \frac{g_{c_2} d_2}{q_5^2} + \frac{e_{s_2} d_2}{q_5^2}\right] = 0 \qquad (23)$$

$$\frac{\partial L}{\partial q_6} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_1 d_1}{q_6^2} - \frac{g_{c_1} d_1}{q_6^2} - \frac{e_{s_1} d_1}{q_6^2}\right] = 0 \qquad (24)$$

$$\frac{\partial L}{\partial \lambda_1} = 0$$

$$\Rightarrow -(q_2 - q_1) = 0,$$

$$\frac{\partial L}{\partial \lambda_2} = 0$$

$$\Rightarrow -(q_3 - q_4) = 0 \text{ and}$$

$$\frac{\partial L}{\partial \lambda_3} = 0$$

$$\Rightarrow -(q_4 - q_5) = 0 \qquad (25)$$

(19) + (20) + (21) +(22) and From the equation (25) we obtain,

$$q_1 = q_2 = q_3 = q_4 = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5) + 2(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4) + 2(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3)]}{s_1 + 3s_2 + 2s_3 + 2s_4}}$$

$$q_5 = \sqrt{\frac{2[(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2)]}{(s_5)}},$$

$$q_6 = \sqrt{\frac{2[(w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{(s_6)}}$$

$$q_1 = q_2 = q_3 = q_4 > q_5 > q_6.$$

It does not satisfy the local optimum

Step 5:

Set k = 4(fix the constraints as 4)

Convert the inequality constraints $q_2 - q_1 \geq 0$, $q_3 - q_2 \geq 0$, $q_4 - q_3 \geq 0$ and $q_5 - q_4 \geq 0$ into an equality constraints $q_3 - q_2 = 0, q_2 - q_1 = 0$, $q_4 - q_3 = 0$ *and* $q_5 - q_4 = 0$.

Minimize F $((T_{\tilde{c}})$by the lagrangean method.

$$L(q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = F(T_{\tilde{c}}) - \lambda_1 (q_2 - q_1) - \lambda_2 (q_3 - q_2) - \lambda_3 (q_4 - q_3) - \lambda_4 (q_5 - q_4) \qquad (26)$$

Differentiate (26) partially with respect to $q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and equate them to zero.

$$\frac{\partial L}{\partial q_1} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_6 d_6}{q_1^2} - \frac{g_{c_6} d_6}{q_1^2} - \frac{e_{s_6} d_6}{q_1^2}\right] + \lambda_1 = 0 \qquad (27)$$

$$\frac{\partial L}{\partial q_2} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_5}{2} - \frac{w_5 d_5}{q_2^2} + \frac{g_{c_5} d_5}{q_2^2} + \frac{e_{s_5} d_5}{q_2^2}\right] - \lambda_1 + \lambda_2 = 0 \qquad (28)$$

$$\frac{\partial L}{\partial q_3} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_4}{2} - \frac{w_4 d_4}{q_3^2} - \frac{g_{c_4} d_4}{q_3^2} - \frac{e_{s_4} d_4}{q_3^2}\right] - \lambda_2 + \lambda_3 = 0 \qquad (29)$$

$$\frac{\partial L}{\partial q_4} = 0$$

$$\Rightarrow \frac{2}{12}\left[\frac{s_3}{2} - \frac{w_3 d_3}{q_4^2} - \frac{g_{c_3} d_3}{q_4^2} - \frac{e_{s_3} d_3}{q_4^2}\right] - \lambda_3 + \lambda_4 = 0 \qquad (30)$$

$$\frac{\partial L}{\partial q_5} = 0$$

$$\Rightarrow \frac{3}{12}\left[\frac{s_2}{2} - \frac{w_2 d_2}{q_5^2} - \frac{g_{c_2} d_2}{q_5^2} + \frac{e_{s_2} d_2}{q_5^2}\right] - \lambda_4 = 0 \qquad (31)$$

$$\frac{\partial L}{\partial q_6} = 0$$

$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_1 d_1}{q_6^2} - \frac{g_{c_1} d_1}{q_6^2} - \frac{e_{s_1} d_1}{q_6^2}\right] = 0 \qquad (32)$$

$$\frac{\partial L}{\partial \lambda_1} = 0$$
$$\Rightarrow -(q_2 - q_1) = 0,$$
$$\frac{\partial L}{\partial \lambda_2} = 0$$
$$\Rightarrow -(q_3 - q_4) = 0,$$
$$\frac{\partial L}{\partial \lambda_3} = 0\ 0$$
$$\Rightarrow -(q_4 - q_5) = 0$$
and $\frac{\partial L}{\partial \lambda_4} = 0$
$$\Rightarrow -(q_5 - q_4) = 0 \qquad (33)$$

(27) + (28) + (29)+(30)+(31) and From the equation (33) we get

$$q_1 = q_2 = q_3 = q_4 = q_5 = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5) + 2(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4) + 2(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3) + 3(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2]}{s_1 + 3s_2 + 2s_3 + 2s_4 + 3s_5}}$$

$$q_6 = \sqrt{\frac{2[(w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{(s_6)}}$$

$$q_1 = q_2 = q_3 = q_4 = q_5 > q_6.$$

It does not satisfy the local optimum

Step 6:

Set k = 5(fix the constraints as 5)

Convert the inequality constraints $q_2 - q_1 \geq 0$, $q_3 - q_2 \geq 0$, $q_4 - q_3 \geq 0$, $q_5 - q_4 \geq 0$ and $q_6 - q_5 \geq 0$ into an equality constraints $q_3 - q_2 = 0$, $q_2 - q_1 = 0$, $q_4 - q_3 = 0$, $q_5 - q_4 = 0$ and $q_6 - q_5 = 0.$

Minimize F $((T_{\tilde{c}})$by the lagrangean method.

$$L(q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) = F(T_{\tilde{c}}) - \lambda_1 (q_2 - q_1) - \lambda_2 (q_3 - q_2) - \lambda_3 (q_4 - q_3) - \lambda_4 (q_5 - q_4) - \lambda_5 (q_6 - q_5) \qquad (34)$$

Differentiate (34) partially with respect to $q_1, q_2, q_3, q_4, q_5, q_6, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and equate them to zero.

$$\frac{\partial L}{\partial q_1} = 0$$
$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_6 d_6}{q_1^2} - \frac{g_{c_6} d_6}{q_1^2} - \frac{e_{s_6} d_6}{q_1^2}\right] + \lambda_1 = 0 \qquad (35)$$
$$\frac{\partial L}{\partial q_2} = 0$$
$$\Rightarrow \frac{3}{12}\left[\frac{s_5}{2} - \frac{w_5 d_5}{q_2^2} + \frac{g_{c_5} d_5}{q_2^2} + \frac{e_{s_5} d_5}{q_2^2}\right] - \lambda_1 + \lambda_2 = 0 \qquad (36)$$
$$\frac{\partial L}{\partial q_3} = 0$$
$$\Rightarrow \frac{2}{12}\left[\frac{s_4}{2} - \frac{w_4 d_4}{q_3^2} - \frac{g_{c_4} d_4}{q_3^2} - \frac{e_{s_4} d_4}{q_3^2}\right] - \lambda_2 + \lambda_3 = 0 \qquad (37)$$
$$\frac{\partial L}{\partial q_4} = 0$$
$$\Rightarrow \frac{2}{12}\left[\frac{s_3}{2} - \frac{w_3 d_3}{q_4^2} - \frac{g_{c_3} d_3}{q_4^2} - \frac{e_{s_3} d_3}{q_4^2}\right] - \lambda_3 + \lambda_4 = 0 \qquad (38)$$
$$\frac{\partial L}{\partial q_5} = 0$$
$$\Rightarrow \frac{3}{12}\left[\frac{s_2}{2} - \frac{w_2 d_2}{q_5^2} - \frac{g_{c_2} d_2}{q_5^2} + \frac{e_{s_2} d_2}{q_5^2}\right] - \lambda_4 + \lambda_5 = 0 \qquad (39)$$
$$\frac{\partial L}{\partial q_6} = 0$$
$$\Rightarrow \frac{1}{12}\left[\frac{s_1}{2} - \frac{w_1 d_1}{q_6^2} - \frac{g_{c_1} d_1}{q_6^2} - \frac{e_{s_1} d_1}{q_6^2}\right] - \lambda_5 = 0 \qquad (40)$$
$$\frac{\partial L}{\partial \lambda_1} = 0$$
$$\Rightarrow -(q_2 - q_1) = 0,$$
$$\frac{\partial L}{\partial \lambda_2} = 0$$
$$\Rightarrow -(q_3 - q_4) = 0,$$
$$\frac{\partial L}{\partial \lambda_3} = 0$$
$$\Rightarrow -(q_4 - q_5) = 0,$$

$$\frac{\partial L}{\partial \lambda_4} = 0$$

$$\Rightarrow -(q_5 - q_4) = 0 \quad \text{and}$$

$$\frac{\partial L}{\partial \lambda_5} = 0$$

$$\Rightarrow -(q_6 - q_5) = 0 \qquad (41)$$

(35) + (36) + (37)+(38)+(39)+(40) and From the equation (41) we get

$$q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5) + 2(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4) + 2(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3) + 3(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2) + (w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{s_1 + 3s_2 + 2s_3 + 2s_4 + 3s_5 + s_6}}$$

$$q^* = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5) + 2(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4) + 2(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3) + 3(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2) + (w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{s_1 + 3s_2 + 2s_3 + 2s_4 + 3s_5 + s_6}}$$

6. Numerical Example:

W = 750, G_c = 60, E_s = 15, D= 3100, M_c = 4000, S =20

Solution:

The Optimal solution is derived for both crisp and fuzzy way.

Crisp Sense:

$$q = \sqrt{\frac{2(W + G_c + E_s)}{S}} = 506$$

Total inventory cost (crisp):

$$T_c = \frac{WD}{q} + M_c(D) + \frac{Sq}{2} + \frac{G_c D}{q} + \frac{E_s D}{q} = 1, 24, 10, 114.35$$

Fuzzy Sense:

$$\tilde{W} = (748.8, 749, 749.5, 750.5, 751, 751.2),$$
$$\tilde{G_c} = (58.8, 59, 59.5, 60.5, 61, 61.2),$$
$$\tilde{E_s} = (13.8, 14, 14.5, 15.5, 16, 16.2),$$
$$\tilde{D} = (3098.8, 3099, 3099.5, 3100.5, 3101, 3101.2)$$
$$\tilde{M_c} = (3998.8, 3999, 3999.5, 4000.5, 4001, 4001.2) \text{ and}$$
$$\tilde{S} = (18.8, 19, 19.5, 20.5, 21, 21.2)$$

$$q^* = \sqrt{\frac{2[(w_6 d_6 + g_{c_6} d_6 + e_{s_6} d_6) + 3(w_5 d_5 + g_{c_5} d_5 + e_{s_5} d_5) + 2(w_4 d_4 + g_{c_4} d_4 + e_{s_4} d_4) + 2(w_3 d_3 + g_{c_3} d_3 + e_{s_3} d_3) + 3(w_2 d_2 + g_{c_2} d_2 + e_{s_2} d_2) + (w_1 d_1 + g_{c_1} d_1 + e_{s_1} d_1)]}{s_1 + 3s_2 + 2s_3 + 2s_4 + 3s_5 + s_6}} = 506$$

Total inventory cost (fuzzy sense):

$T_{\tilde{C}}$ = {12401268.18, 12402742.34, 12406428.09, 12413801.10, 12417488.36, 12418963.41}

$$F(T_{\tilde{C}}) = \frac{1}{12}\left[\begin{array}{c} \left(\frac{w_1 d_1}{q_{6,}} + m_{c_1} d_1 + \frac{s_1 q_{1,}}{2} + \frac{g_{c_1} d_1}{q_{6,}} + \frac{e_{s_1} d_1}{q_{6,}}\right) + \\ 3\left(\frac{w_2 d_2}{q_5} + m_{c_2} d_2 + \frac{s_2 q_2}{2} + \frac{g_{c_2} d_2}{q_5} + \frac{e_{s_2} d_2}{q_5}\right) + \\ 2\left(\frac{w_3 d_3}{q_4} + m_{c_3} d_3 + \frac{s_3 q_3}{2} + \frac{g_{c_3} d_3}{q_4} + \frac{e_{s_3} d_3}{q_4}\right) + \\ 2\left(\frac{w_4 d_4}{q_3} + m_{c_4} d_4 + \frac{s_4 q_4}{2} + \frac{g_{c_4} d_4}{q_3} + \frac{e_{s_4} d_4}{q_3}\right) + \\ 3\left(\frac{w_5 d_5}{q_2} + m_{c_5} d_5 + \frac{s_5 q_5}{2} + \frac{g_{c_5} d_5}{q_2} + \frac{e_{s_5} d_5}{q_2}\right) + \\ \left(\frac{w_6 d_6}{q_1} + m_{c_6} d_6 + \frac{s_6 q_6}{2} + \frac{g_{c_6} d_6}{q_1} + \frac{e_{s_6} d_6}{q_1}\right) \end{array}\right]$$

= 12409930.92

7. Conclusion:

The importances of the green manufacturing inventory model are discussed. This study is a true combination of economic and environmental factors that benefits both the manufacturing and environmental sectors. In this paper, both a precise and hazy manner of the problem is given. Graded mean integration representation method is used to defuzzified the data. The optimum order quantity and the total cost were determined in both crisp and fuzzy environment. The final formulation has been verified using numerical illustrations in both the senses. Hence these inventory models can be functional and useful in the real world.

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