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PROPERTIES OF γ - PARACOMPACT SPACES

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Abstract:

A kind of new paracompactness axiom is introduce in L-topological spaces, where L is a fuzzy lattice. And its topological properties are systematically studied.

1. Introduction:

It is known that paracompactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [1], various kinds of fuzzy compactness [1, 4, 10] have been established. However, these concepts of fuzzy compactness rely on the structure of L and L is required to be completely distributive. In [9], for a complete De Morgan algebra L, Shi introduced a new definition of fuzzy compactness in L – topological spaces using open L – sets and their inequality. This new definition does not depend on the structure of L. In this paper, A kind of new paracompactness axiom is introduced in L – topological spaces, where L is a fuzzy lattice. And its topological properties are systematically studied.

2. Preliminaries:

Throughout this paper X and Y will be nonempty ordinary sets and L = L(<, ∨, ∧') will denote a fuzzy lattice, that is, a completely distributive lattice with a smallest element 0 and largest element 1 ($0 \neq 1$) and with an order reversion involution $a \to a'$ (a ε L). We shall denote by L^X the lattice of all L – subsets of X and if A ε X by X_A the characteristic function of A. An L-topological space is a pair (X, $\mathcal{T}$), where $\mathcal{T}$ is a sub family of LX which contains $\underline{0}$, $\underline{1}$ and is closed for any suprema and finite infima. $\mathcal{T}$ is called an open L – set and its quasi complementation is called a closed L – set. An element p of L is called prime if, and only if $p \neq 1$ and whenever a, b ε L with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$ [3, 4]. The set of all prime elements of L will be denoted by pr (L). An element α of L is called union irreducible or coprime if, and only if whenever a, b ε L with $\alpha \leq a \vee b$ then $\alpha \leq a$ or $\alpha \leq b$ [3]. The set of all non zero union irreducible elements of L will be denoted by M(L). It is obvious that p ε pr (L) if, and only if p' ε M(L). Warner [12] has determined the prime element of the fuzzy lattice L^X. We have pr (L^X) = {x_p : x ε X and p ε pr (L)}, where for each x ε X and each p ε pr (L), x_p : $X \to L$ is the L – subset defined by

$$x_p(y) = \begin{cases} p & \text{if } y=x, \\ 1 & \text{Otherwise.} \end{cases}$$

These x_p are called the L-points of X and we say that x_p is a member of an L-subset f and write $x_p \in f$ if, and only if $f(x) \not\leq p$. Thus, the union irreducible elements of Lx are the function x_a: $X \to L$ defined by,

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } y=x, \\ 0 & \text{Otherwise} \end{cases}$$

Where $x \in X$ and $a \in M(L)$. Hence, we have $M(L^x)=\{x_\alpha : x \in X$ and $a \in M(L))$. As these x_{α}- are identified with the L-points x_p of X,we shall refer to them as fuzzy points. When $x_\alpha \in M(L^x)$, we hall x and α the support of $x_\alpha(x=Suppx_\alpha)$ and the height of $x_\alpha(\alpha=h(x_\alpha))$, respectively.

Definition 2.1 [5]: Let (X,τ) be an L-topological space, $A \in L^x$. Then A is called a γ–open set if A≤Int(Cl)A))vCl(Int(A)). The complement of a γ-open set is called a γ- closed set. Also, $\gamma O(L^x)$ and $\gamma C(L^x)$ will always denote the family of all γ- open sets and γ- closed sets respectively. Obviously, $A \in \gamma O(L^x)$ if , and only if $A' \in \gamma O(L^x)$.

Definition 2.2 [5]: Let (L^x,τ) be an L-topological space, A, $B \in L^x$. Let $\gamma Int(A)=V\{B \in L^X \backslash B \leq A, B \in \gamma O(L^x)\}$, $\gamma Cl(A)= \wedge \{B \in L^X \backslash A \leq B, B \in \gamma Cl(L^x)\}$. Then γ Int(a) and γCl(A) are called the γ -interior and γ -closure of A respectively.

Definition 2.3 [5]: Let (X,τ) and (Y,σ) be two L-topological spaces. A function $f(X,\tau) \to (Y,\sigma)$ is called γ–continuous if, and only if $f^{-1}(g)$ is γ-open in (X,τ), for each $g \in \sigma$.

Definition 2.4: Let $\alpha \in M(L)$ and $g \in L^x$, A collection η of L-subsets is said to form an α –level filter base in the L-subset g if, and only if for any finite subcollection $\{f_1,\ldots\ldots f_n\}$of η, there exists $x \in X$. with $g(x) \geq \alpha$ such that

$(\bigwedge_{i=1}^{n} f_i)(x) \geq \alpha$. When g is the whole space X, then η is an α –lever filter base if, and only if for any finite subcollection$\{f_1,\ldots\ldots f_n\}$of η,t here exists $x \in X$ such that $(\bigwedge_{i=1}^{n} f_i)(x) \geq \alpha$.

Lemma 2.5 [9]: Let (X,τ) be a topological space, f be an L-subset in the L-ts ((X,ω(τ)) and $p \in pr(L)$. Then we have

1. $(Cl(f))^{-1}(\{t \in L : t \not\leq p\}) \subset (Cl(f^{-1}(\{t \in L : t \not\leq p\}))$
2. $(Int(f))^{-1}(\{t \in L : t \not\leq p\}) \subset (Cl(f^{-1}(\{t \in L : t \not\leq p\}))$

Lemma 2.6 [9]: Let (X,τ) be a topological space and $A \subset X$. Considering the L-ts (X, ω(τ)) and

$$f(X) = \begin{cases} e \in L & \text{if } x \text{ A}, \\ 0 & \text{otherwise}, \end{cases}$$

We have the following

$$Cl(f)(X) = \begin{cases} e & \text{if } x \in Cl(A), \\ 0 & \text{otherwise}, \end{cases}$$

and

$$Int(f)(X) = \begin{cases} e & \text{if } x \in Int(A), \\ 0 & \text{otherwise}, \end{cases}$$

Definition 2.7 [6]: Let (X,τ) be an L-ts and $g \in L^x$, $r \in L$.

1. A collection $\mu = \{f_i\}_{i \in J}$ of L-subsets is called an r-level cover of g if, and only if $(\bigvee_{i \in J} f_i)(x) \not\leq r$ for all $x \in X$ with g(x)r'. If each f_i is open then μ is calledan r-level open cover of g. If g is the whole space X, then μ is called an r-level cover of X if, and only if $(\bigvee_{i \in J} f_i)(x) \not\leq r$ for all $x \in X$.
2. An r-level cover $\mu = \{f_i\}_{i \in J}$ of g is said to have a finite r-level subcover if there exists a finite subset F of J such that $(\bigvee_{i \in J} f_i)(x) \not\leq r$ for all $x \in X$ with $g(x) \geq r'$

Definition 2.8: Let (X,τ) be an L-ts and $g \in L^x$. Then g is said to be compact [7] if, and only if for every prime $p \in L$ and every collection $\{f_i\}_{i \in J}$ of open L-subsets with $(\bigvee_{i \in J} f_i)(x) \not\leq r$ for all $x \in X$ with $g(x) \geq p'$, there exists a finite subset F of J such that $(\bigvee_{i \in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$, that is, every p-level open cover of g has a finite p-level subcover, where $p \in pr(L)$. If g is the whole space, then the L-ts (X,τ) is called compact.

3. γ- Compactness and Its Goodness:

Definition 3.1: Let (X,τ) be an L-topological space and $g \in L^x$. The g is called γ-compact if, and only if every p-level cover of g consisting of γ -open L-subsets has a finite p-level subcover, where $p \in pr(L)$. If g is the whole space, then we say that the L-topological space (X,τ) is γ–compact.

Lemma 3.2: Let (X,τ) be an topological space and $A \subset X$. If A is γ-open in (X,τ) , then χ_A is γ-open in the L-topological space (X,ω (ι)).

Proof: The proof is clear.

Theorem 3.3: Let (X,τ) be a topological space. Then (X,τ) is γ-compact if, and only if the L-topological space (X,ω (ι)) is γ–compact.

Proof: Let $p \in pr(L)$ and $\{f_i\}_{i \in J}$ be a p-level γ-open cover of (X,ω (ι)). Then $(\bigvee_{i \in J} f_i)(x) \not\leq p$ for all $x \in X$. Hence for each $x \in X$ there is $i \in J$ such that $f_i(x) \not\leq p$, that is , $x \in f_i^{-1}(\{t \in L : t \not\leq p\})$. So, $X = \bigcup_{i \in J} f_i^{-1}(\{t \in L : t \not\leq p\})$. Because f_i is γ- open in (X,ω(ι)), $f_i^{-1}(\{t \in L : t \not\leq p\})$ is γ- open in (X,τ). Thus $f_i^{-1}(\{t \in L : t \not\leq p\})\}_{i \in j}$ is a γ-open cover of (X,τ). Since (X,τ) is γ-compact, there is a finite subset F of J such that $X = \bigcup_{i \in J} f_i^{-1}(\{t \in L : t \not\leq p\})$, that is $(\bigvee_{i \in J} f_i)(x) \not\leq p$ for all $x \in X$. Hence (X,ω(ι)) is γ-compact. Conversely let $\{A_i\}_{i \in J}$ be a γ-open cover of (X,τ). Then by Lemma 3.2 $\{X_{Ai}\}_{i \in J}$ is a family of γ-open L-subsets in (X,ω(ι₁)), ω(ι₂)) such that $1 = (\bigvee_{i \in J} X_{Ai})(x) \not\leq p$ for all $x \in X$ and for all $p \in pr(L)$, that is $\{X_{Ai}\}_{i \in J}$ is a p-level γ-open cover of (X,ω(ι)). Since (X,ω(ι)) is γ-compact, there is a finite F of J such that $(\bigvee_{i\in J} X_{Ai})(x)\not\leq p$ for all $x\in X$. Hence $(\bigvee_{i\in F} X_{Ai})(x)=1$ for all $x\in X$, that is, $X=\bigcup_{i\in F} A_i$ and therefore, is (X,τ)–compact.

Theorem 3.4: Let (X,τ) be an L-topological space. Then $g\in L^x$ is γ- compact if , and only if for every $\alpha\in M(L)$ and every collection $\{h_i\}_{i\in J}$ of γ- closed L-subsets with $(\bigwedge_{i\in J} h_i)(x)\not\geq \alpha$ for all $x\in X$ with $g(x)\geq\alpha$, there is a finite subset F of J such that $(\bigwedge_{i\in J} h_i)(x)\not\geq \alpha$ for all $x\in X$ with $g(x)\geq\alpha$.

Proof: This follows immediately from Definition 3.1

Theorem 3.5 Let (X,τ) be an L-topological space. Then $g\in L^x$ is γ- compact if , and only if for every $p\in pr(L)$ and every collection $\{f_i\}_{i\in J}$ of γ- open L-subsets with $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$, there is a finite subset F of J such that $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$.

Proof: Let $p\in pr$ (L) and $\{f_i\}_{i\in J}$ be a collection of γ-open L-subsets with $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$. Then $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$ with $g(x)\not\geq p'$. Since g is γ-compact, there is a finite subset F of J such that $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$ with $g(x)\not\geq p'$. Take an arbitrary $x\in X$. If $g'(x)\not\leq p$, then $g'(x) v (\bigvee_{i\in F} f_i(x) =(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ because $(\bigvee_{i\in J} f_i(x)\not\leq p$, If $g'(x)\not\leq p$, then we have $g'(x)$ v $(\bigvee_{i\in F} f_i(x) =(\bigvee_{i\in J} f_i v g')(x)\not\leq p$, Thus, we have $(\bigvee_{i\in F} f_i v g')(x)\not\leq p$ for all $x\in X$. Conversely, let $p\in pr$ (L) and $\{f_i\}_{i\in J}$ be a collection of a p-level, γ-open cover of g. Then $(\bigvee_{i\in J} f_i)(x)\not\leq p$ for all $x\in X$. with $g(x)\not\geq p'$. Hence $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$. From the hypothesis, there is a finite subset F of J such that $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$. Then $(\bigvee_{i\in J} f_i v g')(x)\not\leq p$ for all $x\in X$ with $g'(x)\not\leq p'$. Thus g is γ-compact.

Definition 3.6: Let (X,τ) be an L-topological space. X_α be an L-point in $M(L^x)$ and $S=(S_m)_{m\in D}$ be a net, X_α is called γ-cluster point of S if, and only if for each γ- closed L-subset f with $f(x)\not\geq\alpha$ and for all $n\in D$, there is $m\in D$ such that $m\geq n$ and $S_m\not\leq f$, that is, $h(S_m)\not\leq f(SuppS_m)$.

Theorem 3.7: Let (X,τ) be an L-topological space. Then $g\in L^x$ is γ-compact if, and only if every constant α net in g, where $a\in M(L)$, has a γ-cluster point in g with height α.

Proof: Let $\alpha\in M(L)$ and S= $S=(S_m)_{m\in D}$ be a constant α –net in g without any γ-cluster point with height α in g. Then for each $x\in X$ with $g(x)\not\geq\alpha X_\alpha$ is not a γ-cluster point of S, that is, there are $n_x\in D$ and a γ-closed, L-subset fx with $f_x(x)\not\geq\alpha$ and $S_m\leq f_x$ for each $m\geq n_x$. Let $x_1,\ldots\ldots.x_k$ be elements of X with $g(x^i)\geq a$ for each $i\in\{l,...k\}$. Then there are $nx_1,...,n_{xk}\in D$ and γ-closed L-subset f_{xi} with $f_i(x^i)\geq\alpha$ and $S_m\leq f_{xi}$ for each $m\geq n_{xi}$ and for each $i\in\{l,...k\}$.Since D is a directed set, there is $n_0\in D$ such that $n_0\geq n_{xi}$ for each $i\in\{l,...k\}$ and $S_m\leq f_{xi}$ for $i\in\{l,...k\}$ and each $m\geq n_0$. Now, consider the family $\mu=\{fx\}_{x\in X}$ with $g(x)\geq\alpha$. Then $(\bigwedge_{fx\in\mu} f_x)(y)\not\geq\alpha$ for all $y\in X$ with $g(y)\geq\alpha$ because $f_y(y)\not\geq\alpha$.We also have that for any finite subfamily $\upsilon=\{fx_1,\ldots\ldots fx_k\}$ of μ, there is $y\in X$ with $g(y)\geq\alpha$ and $(\bigwedge_{i=1}^{k} f_{xi})(y)\not\geq\alpha$ since $S_m\leq \bigwedge_{i=1}^{k} f_{xi}$ $m\geq n_0$ because $S_m\leq f_{xi}$ for each $m\geq n_0$.

Hence, by Theorem 3.5, g is not γ-compact. Conversely, suppose that g is not γ-compact. Then by Theorem 3.5, there exist $\alpha\in M(L)$ and a collection $\mu=\{f_i\}_{i\in J}$ of γ- closed L-subsets with $(\bigwedge_{i\in J} f_i)(x)\not\geq\alpha$ for all $x\in X$ with $g(x)\not\geq\alpha$, but for any finite subfamily υ of μ there is $x\in X$ with $g(x)\not\geq\alpha$ and $(\bigwedge_{i\in J} f_i)(x)\geq\alpha$ .Consider the family of all finite subsets of μ, $2^{(\mu)}$, with the order $\upsilon_1\leq\upsilon_2$ if, and only if $\upsilon_1\subset\upsilon_2$. Then $2^{(\mu)}$ is a directed set. So, writing x_α as S_υ for every $\upsilon\in 2^{(\mu)}$,$(X_\upsilon)_{\upsilon\in 2(\mu)}$ is a constant α –net in g because the height of S_υ for all $\upsilon\in 2^{(\mu)}$ is α and $S_\upsilon \le g$ for all $\upsilon\in 2^{(\mu)}$, that is $g(x) \not\geq \alpha$. $(S_\upsilon)_{\upsilon\in 2(\mu)}$ also satisfies the condition that for each γ-closed L-subset, $f_i\in\upsilon$ we have $x_\alpha = S_\upsilon \le f_i$, . Let $y\in X$ with $g(y)\ge\alpha$. Then $(\bigwedge_{i\in J} f_i)(y)\ge \alpha$, that is, there exists $j\in J$ with $f_j(y)\ge \alpha$. Let $\upsilon_0 = \{f_j\}$. So, for any $\upsilon\ge\upsilon_0$, $S_\upsilon\le \bigwedge_{fi\in\upsilon} f_i \le \bigwedge_{fi\in\upsilon} f_{i=}f_{i.}$ Thus, we get a γ- closed L-subset. f_j with $f_j(y)\ge \alpha$ and $\upsilon_0 \in 2^{(\mu)}$ such that for any $\upsilon\ge\upsilon_0$, $S_\upsilon\le f_i$. That means that $y_\alpha\in M(L^X)$ is a not a γ- cluster point $(X_\upsilon)_{\upsilon\in 2(\mu)}$ for all $y\in X$ with $g(y)\ge \alpha$. Hence, the constant α-net $(S_\upsilon)_{\upsilon\in 2(\mu)}$ has no γ- cluster point in g with height α.

Corollary 3.8: An L-topological space is (X,τ) γ-compact if, and only if every constant α-net in (X,τ) has a γ-cluster point with height α, where $\alpha\in M(L)$

Definition 3.9: Let (X,τ) be an L-topological space and η an α –level filter base, where $\alpha\in M(L)$. An L-point $x_r\in M(L^x)$ is called a γ-cluster point of η, if $\bigwedge_{f\in\eta} \gamma Cl(f)(x)\ge r$.

Theorem 3.10: Let (X,τ) be an L-topological space. Then $g\in L^x$ is γ-compact if, and only if every α –filter base in g, where $\alpha\in M(L)$, $\alpha\gamma$-cluster point x_α in g with height α.

Proof: Assume that η is an α –level filter base in g with no γ-cluster point in g with height α, where $\alpha\in M(L)$. Then for each $x\in X$ with $g(x)\ge \alpha$,x_α is not a γ-cluster point of η , that is, there is $f_x\in\eta$ with $\gamma Cl(f)(x)\not\geq \alpha$. Hence $\gamma Cl(f_x)'(x)\not\geq \alpha=p\in pr(L)$.This means that the collection $\{\gamma Cl(f_x)'\}_{x\in X}$ with $g(x)\ge \alpha$ is a p–level γ- open cover of g. Since g is γ–compact, there $\gamma Cl(f_{x1}),\ldots\ldots,\gamma Cl(f_{xn})$ such that $\bigvee_{i=1}^{n} \gamma Cl(f_{xi})'(x)\not\leq p$ for all $x\in X$ with $g(x) \ge p' = \alpha$. Hence $\bigwedge_{i=1}^{n} \gamma Cl(f_{xi})'(x)\not\geq\alpha$ for all $x\in X$ with $g(x) \ge \alpha$ which implies that $(\bigwedge_{i=1}^{n} f_{xi})(x)\not\geq\alpha$ for all $x\in X$ with $g(x) \ge \alpha$. This is a contradiction. Conversely, suppose that g is not γ-compact. Then there is a p-level γ-open cover μ of g with no finite p-level subcover, where $p\in pr(L)$. Hence for each finite sub collection $\{h_1,\ldots\ldots h_n\}$of μ, there exists $x\in X$ with $g(x) \ge p'$ such that $(\bigvee_{i=1}^{n} (h_i(x)\le p$, that is, $(\bigvee_{i=1}^{n} (h_i(x) \ge p' = \alpha\in M(L)$. Thus $\eta=\{h{:}h\in\mu\}$ forms an α –level filter base in g. By the hypothesis, μ has a γ-cluster point $y_\alpha\in M(L^x)$ in g with height α, that is, $g(y) \ge \alpha$ and $\bigwedge_{h\in\mu} \gamma Cl(h')(y) = \bigwedge_{h\in\mu} h')(y) \ge \alpha$, Then $\bigwedge_{h\in\mu} h')(y) \le p$, which yields a contradiction.

Corollary 3.11: An L-topological space (X,τ) is γ- compact if, and only if every α –filter base has $\alpha\gamma$-cluster point with height α, where $\alpha\in M(L)$.

Theorem 3.12: Let (X,τ) be an L-topological space and $g,h\in L^x$. If g and h are γ-compact then gVh is γ-compact.

Proof: Let $p\in pr(L)$ and $\{f_i\}_{i\in J}$ be a collections of γ- open L-subsets with $(\bigvee_{i\in J} f_i(x) \not\leq p$ for all $x\in X$ with $(gVh(x)\ge p'$. Since p is prime, we have $(gvh)(x)\ge p'$ if, and only if $g(x)\ge p'$ or $h(x)\ge p'$. So, by the γ–compactness of g and h, there are finite subsets E,F of J such that $(\bigvee_{i\in J} f_i(x) \not\leq p$ for all $x\in X$ with $g(x)\ge p'$ and $(\bigvee_{i\in F} f_i(x) \not\leq p$ for all $x\in X$ with $h(x)\ge p'$. Then $(\bigvee_{i\in EUF} f_i(x) \not\leq p$ for all $x\in X$ with $h(x)\ge p'$ or $h(x)\ge p'$, that is, $(\bigvee_{i\in EUF} f_i(x) \not\leq p$ for all $x\in X$ with $h(x)\ge p'$ or $h(x)\ge p'$. Thus g v h is γ- compact.

Theorem 3.13: Let (X,τ) be an L-topological space and $g,h\in L^x$. If f is γ-compact and h is γ-closed, then $g\wedge h$ is γ–compact.

Proof: Let $p\in pr(L)$ and $\{f_i\}_{i\in J}$ be a collections of γ- open L-subsets with $(\bigvee_{i\in J} f_i(x) \not\leq p$ for all $x\in X$ with $(g\wedge h(x)\ge p'$. Thus $\mu= \{f_i\}_{i\in J}\cup\{h'\}$ is a family of γ-open L-subsets with $(\bigvee_{k\in\mu} k(x) \not\leq p$ for all $x\in X$ with $g(x)\ge p'$.

In fact, for each x∈X with g(x)≥p', if h(x)≥p', then (g∧h(x)≥p' which implies that ($\bigvee_{i\in J}$ f_i(x) ⋠p, thus ($\bigvee_{k\in\mu}$ k(x) ⋠p. If h(x)≱p', then h'(x)⋠p which implies ($\bigvee_{k\in\mu}$ k)(x) ⋠p . From the γ—compactness of g there is a finite subfamily υ of μ say υ= {f_1,……..f_n, h'} with ($\bigvee_{k\in\mu}$ k)(x) ⋠p for all x∈X with g(x)≥p'.

Then ($\bigvee_{i=1}^{n}$ f_i(x) ⋠p for all x∈X with (g∧h (x) ≥p'. Hence g∧h is γ–compact.

Corollary 3.14: Let (X,τ) be an γ-compact space and g be a γ-closed, L-subset. Then g is γ-compact.

Theorem 3.15: Let (X,τ) be an L-topological space where X is a finite set. Then (X,τ) is γ-compact.

Proof: Let $\{f_i\}_{i\in J}$ be a p-level γ- open cover of (X,τ) , whre p∈pr(L). Then ($\bigvee_{i\in J}$ f_i(x) ⋠p for all x∈X. Hence, for each x∈X there is i∈J such that x∈f_i^{-1}({t∈T:t⋠p}). Since X is finite subset F of J such that X= $\bigcup_{i\in F}$ f_i^{-1}({t∈T:t⋠p}), that is, ($\bigvee_{i\in F}$ f_i)(x) ⋠p for each x∈X. Hence (X,τ) is γ-compact.

Corollary 3.16: Let (X,τ) be an L-topological space and g∈L^X . If g with finite support, then g is γ-compact.

Let (X,τ) be an L-topological space. The following δ_ξ will denote the L-topology on X which has the set of all γ-open subsets of (X,τ) as a subbase.

Theorem 3.17: Let. (X, δ) be an L-topological space and g∈$L^{X.}$ Then g is γ-compact in (X,τ) if, and only if g is compact in (X, δ_ξ).

Proof: Let p∈pr(L) and $\{f_i\}_{i\in J}$ be a collection of subbasic δ_ξ-openL-subsets with ($\bigvee_{i\in J}$ f_i(x) ⋠p for all x∈X with g(x)≥p'. Then each f_i is γ-open in (X,τ) and so $\{f_i\}_{i\in J}$ is a p-level γ-open cover of g. Since g is γ-compact in (X,τ) , there is a finite subst F of J such that ($\bigvee_{i\in F}$ f_i(x) ⋠p for each x∈X with g(x)≥p'. Hence g is compact in (X, δ_ξ). Conversely, lep p∈pr(L) and $\{f_i\}_{i\in J}$ be a collection of γ-open L-subsets in (X,τ) with ($\bigvee_{i\in F}$ f_i(x) ⋠p for each x∈X g(x)≥p'. Since every γ-open L-subsets in (X,τ) isδ_ξ –open by the compactness of g in (X, δ_ξ), there is a finite subset F of J such that ($\bigvee_{i\in F}$ f_i(x) ⋠p for each x∈X g(x)≥p'. Hence g is γ-compact in (X,τ).

Corollary 3.18: An L-topological space (X,τ) is γ-compact if, and only if L-topological space (X, δ_ξ) is compact.

Theorem 3.19: Let (X,τ) be an L-topological space. If g is a γ-compact L-subset in (X,τ) then for each closed L-subset h in (X, δ_ξ), h∧g is γ–compact in (X,τ).

Proof: This follows from Theorem 5.6 and 3.4 in [7].

Definition 3.20: Let (X,τ) and (Y,σ) be two L-topological spaces. A function f:(X,τ)→ (Y,σ) is called:

1. ξ-continuous if, and only if f:(X, δ_ξ)→ (Y,σ) is continuous.
2. ξ'-continuous if, and only if f:(X, δ_ξ)→ (Y, ι_ξ) is continuous.

Corollary 3.21: Let (X,τ) and (Y,σ) be two L-topological spaces and f:(X,τ)→ (Y,σ) be a ξ-continuous mapping , then f is ξ—continuous.

Theorem 3.22: Let (X,τ) and (Y,σ) be two L-topological spaces and f:(X,τ)→ (Y,σ) be a ξ-continuous mapping with f^{-1}(y) is finite for every y∈Y. If g∈L^x is γ- compact in(X,τ) , then f(g) is compact in (Y,σ).

Proof: This follows from theorem 5.6 and 3.6 in [7]

Corollary 3.23: Let (X,τ) and (Y,σ) be two L-topological spaces and f:(X,τ)→ (Y,σ) be a γ-continuous mapping with f^{-1}(y) is finite for every y∈Y. If g∈L^x is γ- compact in(X,τ) , then f(g) is compact in (Y,σ).

Corollary 3.24: Let (X,τ) and (Y,σ) be two L-topological spaces and f:(X,τ)→ (Y,σ) be a γ-continuous mapping with f^{-1}(y) is finite for every y∈Y. If (X,τ) is γ- compact then (Y,σ) is compact.

Corollary 3.25: Let (X,τ) and (Y,σ) be two L-topological spaces and f:(X,τ)→ (Y,σ) be a ξ-continuous mapping with f^{-1}(y) is finite for every y∈Y. If g∈L^x is γ– compact in(X,τ) ,then f(g) is γ – compact in (Y,σ).

Proof: This follows from theorem 5.6 and 3.6 in [7].

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