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PRIME LABELING OF TRIANGULAR BOOK AND CYCLE-CACTUS GRAPHS

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Abstract:

In this paper, we prove that the Triangular book B_n and Cycle-Cactus $C_k^{(n)}$ graphs are prime graphs. Here, we investigate and prove that new results for Triangular book B_n admits prime labeling when n is even and odd separately. We also show that the Cycle-Cactus $C_k^{(n)}$ admits prime labeling for all n and k where $k \geq 3$.

Key Words: Prime Labeling and Prime Graphs, Triangular Book & Cycle-Cactus

1. Introduction:

We consider only simple, finite, connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For standard terminology and notations, we follow Bondy and Murty [1]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

Definition 1.1: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 1.2: If the vertices are assigned values subject to certain conditions then it is known as graph labeling.

Definition 1.3: Let $G = (V, E)$ be a graph with n vertices. A bijection $f: V(G) \to \{1,2,..,n\}$ is called a prime labeling if for each edge $e = uv$, $\gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.4: A Triangular book B_n is defined as $B_n = K_2 + \overline{K_t}$

Definition 1.5: The Cycle-Cactus consisting of n copies of cycle c_k , $k \geq 3$ concatenated at exactly one vertex is denoted as $C_k^{(n)}$.

Definition 1.6: The set of vertices adjacent to a vertex u of G is denoted by $N(u)$.

Definition 1.7: (Bertrand's Postulate) For every $n \geq 2$, there exists a prime p such that $n < p < 2n$

The notion of prime labeling was originated by Entringer and it was discussed by Tout et al [8].Fu and Huang [4] proved that P_n and $K_{1,n}$ are prime graphs. Lee et al [7] proved that W_n is a prime graph if and only if n is even. Deretsky et al [3] proved that C_n is a prime graph. In [6], Ganesan and Balamurugan have proved that the Mongolian tent graph is a prime graph. In [9], Vaidya and Kanmani proved the prime labeling for some cycle related graphs. Carlson in [2] has proved that the Generalised Books and C_m snakes are prime graphs.

Remark

Using Bertrand's principle, we can find the value of the prime number p such that either $p = 2n - 1$ or $p = 2n + 1$ or both

2. Main Results:

Theorem 2.1:

The triangular book $B_n = K_2 + \overline{K_t}$ admits prime labeling when n is even.

Proof:

Let $B_n = K_2 + \overline{K_t}$ be a triangular book where t is even.

Let u and v be the vertices of K_2 and $u_1, u_2, \dots, u_t$ be the vertices of $\overline{K_t}$

Then $|V(B_n)| = t + 2$

Define a function $f: V(B_n) \to \{1,2,..,t+2\}$ as follows

For labeling the vertices of B_n ,we consider the following two cases

Case:1 When t is even and $t + 1$ is a prime number

Let $f(u) = 1$ and $f(v) = t + 1$

$f(u_i) = i + 1$, for $1 \leq i \leq t - 1$ and $f(u_t) = t + 2$

Case:2 When t is even and $t + 1$ is not a prime number

Let $f(u) = 1$ and $f(v) = t - 1$

$f(u_i) = i + 1$, for $1 \leq i \leq t - 3$

$f(u_{t-2}) = t$

$f(u_{t-1}) = t+1$
$f(u_t) = t+2$
In view of above defined labeling pattern, f satisfy the condition of prime labeling
Therefore, B_n admits prime labeling
Hence, The triangular book $B_n = K_2 + \overline{K_t}$ is a prime graph.
Illustrations
Case 1: When t is even and $t+1$ is a prime number
(i) Let $t = 6$ and $t+1 = 7$ (prime)

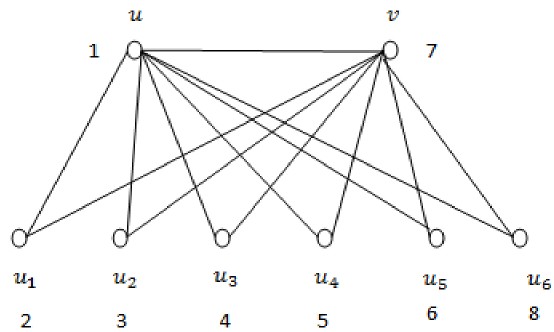

Figure 2.1: Triangular book B_8 and its prime labeling

(ii) Let $t = 10$ and $t+1 = 11$ (prime)

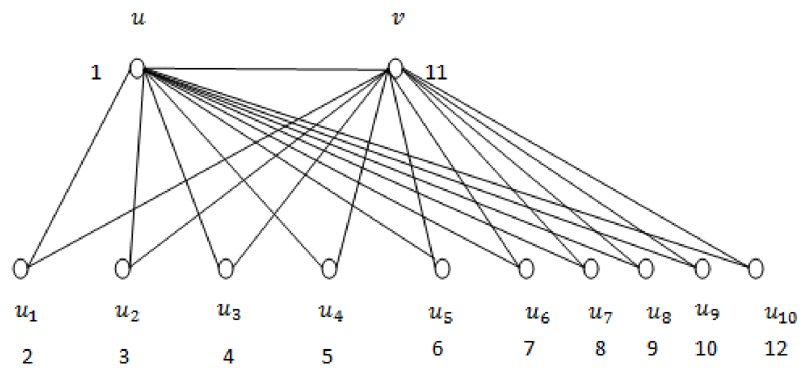

Figure 2.2: Triangular book B_{12} and its prime labeling

Case 2: When t is even and $t+1$ is not a prime number
(i) Let $t = 8$ and $t+1 = 9$ is not a prime

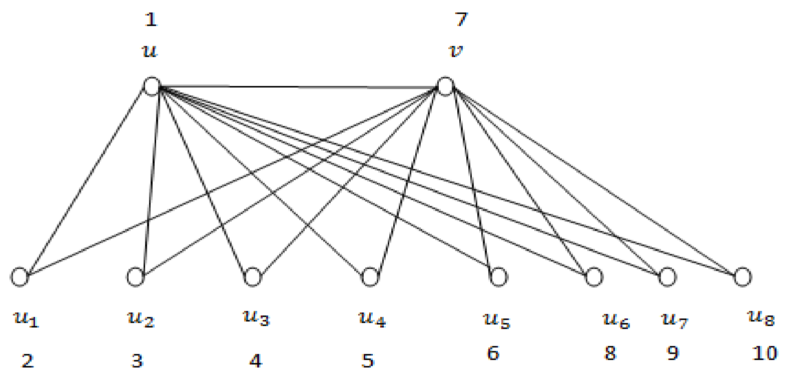

Figure 2.3: Triangular book B_{10} and its prime labeling

(ii) Let $t = 14$ and $t + 1 = 15$ is not a prime

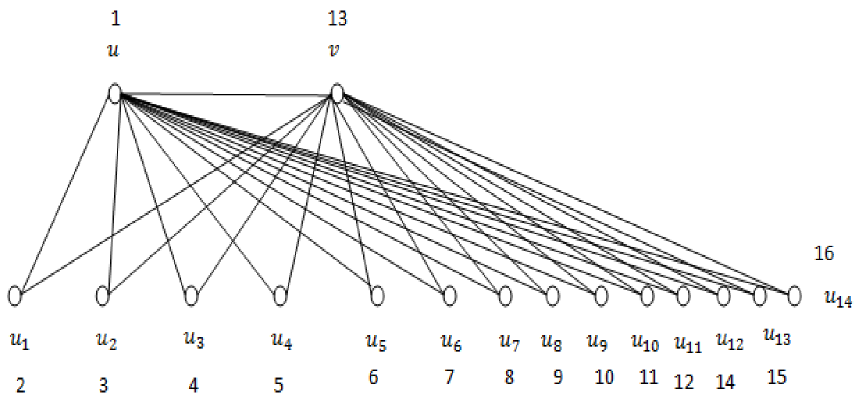

Figure 2.4: Triangular book B_{16} and its prime labeling

Theorem 2.2:

The triangular book $B_n = K_2 + \overline{K_t}$ admits prime labeling when t is odd.

Proof:

Let $B_n = K_2 + \overline{K_t}$ be a triangular book where t is an odd integer

Let u and v be the vertices of K_2 and $u_{1,}u_2, \ldots, u_t$ be the vertices of $\overline{K_t}$

Then $|V(B_n)| = t + 2$= an odd number

Define a function $f: V(B_n) \to \{1,2,.., t + 2\}$ as follows

we consider the following two cases

Case:1 When t is odd and t is a prime number

Let $f(u) = 1$ and $f(v) = t$

$f(u_i) = i + 1$, for $1 \leq i \leq t - 2$

and $f(u_{t-1}) = t + 1$

$f(u_t) = t + 2$

Case:2 When t is odd and t is not a prime number

Let $f(u) = 1$ and $f(v) = t + 2$

$f(u_i) = i + 1$, for $1 \leq i \leq t$

In view of above defined labeling pattern, f satisfy the condition of prime labeling

Hence, the triangular book $B_n = K_2 + \overline{K_t}$ is a prime graph when t is odd

Illustrations:

Case 1: When t is an odd integer and $t = p$ (prime)

Let $t = 7$ and t is prime number

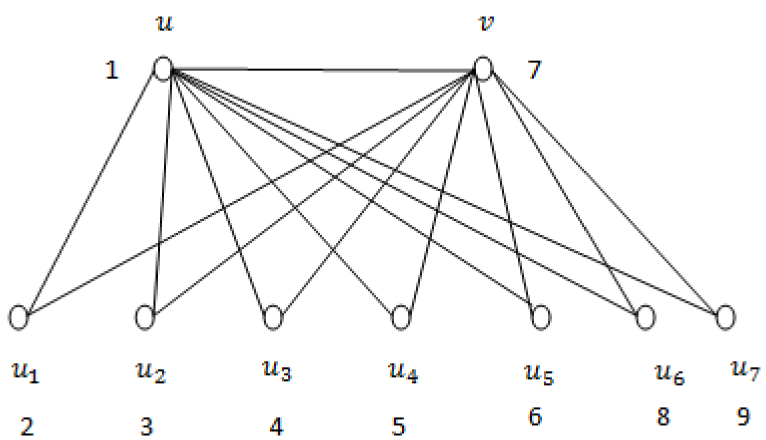

Figure 2.5: Triangular book B_9 and its prime labeling

Case 2: When t is an odd number and $t \neq p$ (prime)
Let $t = 9$ and $t \neq p$

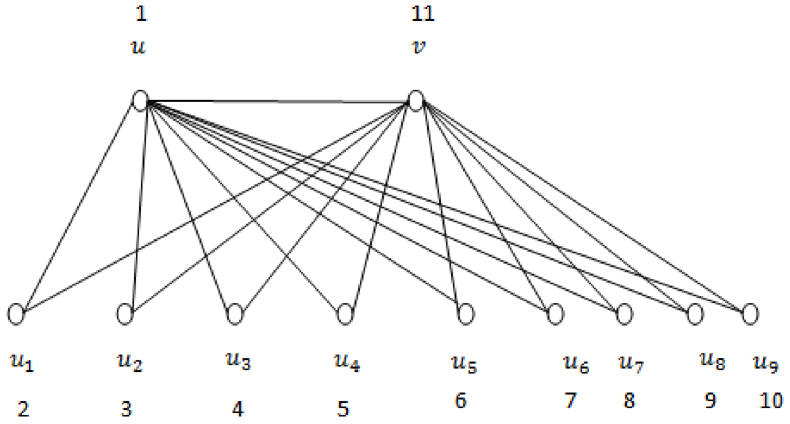

Figure 2.6: Triangular book B_{11} and its prime labeling

Theorem 2.3:

The graph Cycle-Cactus $C_k^{(n)}$ admits prime labeling

Proof:

Let $G = C_k^{(n)}$ be the cycle-cactus graph consisting of n copies of the cycle C_k with k vertices, $k \geq$ 3concatenated at exactly one vertex.

Let $C_k^{(1)}, C_k^{(2)}, \ldots, C_k^{(n)}$ be the n copies of the cycle C_k and these n copies concatenated at the common vertex v. Let $v, v_{12}, v_{13}, \ldots, v_{1k}$ be the vertices of the first copy $C_k^{(1)}$ of the cycle C_k , $v, v_{22}, v_{23}, \ldots, v_{2k}$ be the vertices of the second copy. $C_k^{(2)}$ of the cycle C_k and finally,let $v, v_{n2}, v_{n3}, \ldots, v_{nk}$ be the vertices of the nth copy $C_k^{(n)}$ of the cycle C_k. We assume that all the n copies of the cycle C_k concatenated at the common vertex v.

Then $|V(G)| = n(k-1)+1$

Define a function $f: V(G) \rightarrow \{1,2,..,n(k-1)+1\}$ as follows

Let $f(v) = 1$

$f(v_{1i}) = i+1$, for $2 \leq i \leq k$
$f(v_{2i}) = (k-1)+i$, for $2 \leq i \leq k$
$f(v_{3i}) = (2k-2)+i$, for $2 \leq i \leq k$
$\vdots \qquad \vdots$
$f(v_{ni}) = (n-1)k-(n-1)+i$, for $2 \leq i \leq k$

In view of above defined labeling pattern, f satisfy the condition of prime labeling

Hence, the Cycle-Cactus $C_k^{(n)}$ admits prime labeling.

Illustrations:

(1) Let $k = 5$ and $n = 3$

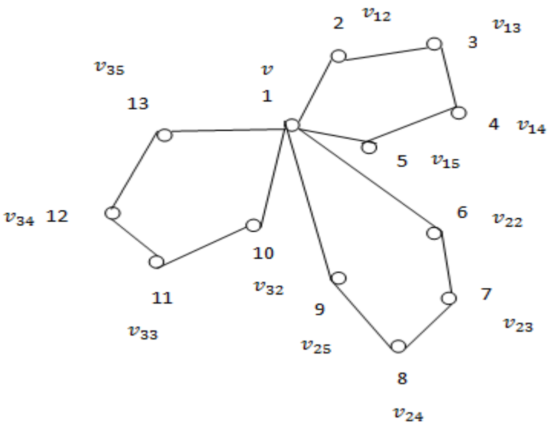

Figure 2.7: The Cycle-Cactus $C_5^{(3)}$ and its prime labeling

(2) Let $k = 4$ and $n = 4$

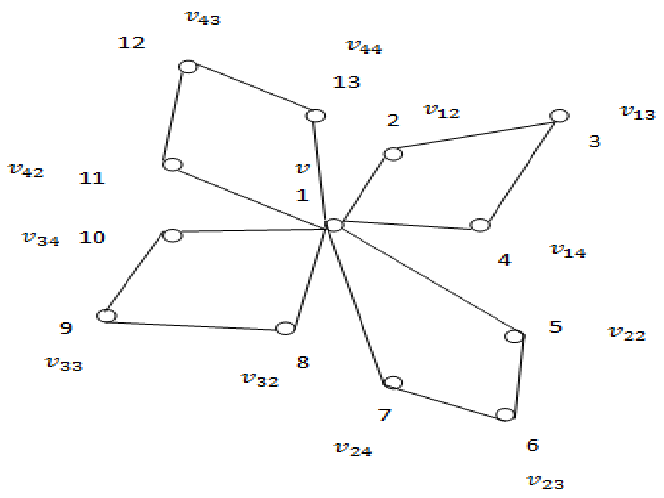

Figure 2.8: The Cycle-Cactus $C_4^{(4)}$ and its Prime Labeling

3. Conclusion:

Labeling of graph is a potential area of research due to its diversified applications and it is very interesting to investigate whether any graph (or) graph family admits a particular labeling or not? Here, we contribute three results in the context of prime labeling of two graphs, namely triangular book and cycle-cactus. Analogous results can be investigated for various graphs and similar results can be obtained in the context of different graph labeling techniques.

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