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AN ADOMIAN DECOMPOSITION METHOD TO SOLVE FUZZY CAUCHY NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract:

In this paper, solving fuzzy Cauchy nonlinear ordinary differential by Adomian decomposition method have been done, and the convergence of the proposed method is proved. This method is illustrated by some numerical examples.

Key Words: Linear Fuzzy Differential Equation, Fuzzy Initial Value Problem & Adomian Decomposition Method

1. Introduction:

The concept of fuzzy derivative was first introduced by Chang et al. [1] it was followed up by Dubois, Prede [2] who defined and used the extension principle. The fuzzy differential equations (FDEs) and the intial value problem where regularly treated by Kaleva et al [3,4] The numerical method for solving fuzzy differential equations is introduced by et al [5] by the standard Eular method. In the last few years many works have been performed by several authours in numerical solutions of fuzzy differential equations Recently, the numerical solution of FDEs by predictor-corrector method has been studied Allahviranloo [3]. In this chapter we replace the fuzzy differential equation by its parametric form and then solve numerically the new system. Which consider the three classic ordinarry differintial equations with initial condition.

2. Preliminaries:

Definition 1 [13]:

Let I be a real interval. A mapping $x : I \to E$ is called a fuzzy process and its r-level set is denoted by $[x(r)]_r = [x_1(t, r), x_2(t, r)]$, $t \in I$ and $r \in (0,1]$. and the derivative $x'(t)$ of a fuzzy process $x(t)$ is defined by $[x'(r)]_r = [x'_1(t, r), x'_2(t,r)], t \in I, r \in (0,1]$

3. Fuzzy Cauchy Problem [9]:

Consider the fuzzy initial value problem

$$\begin{cases} y'(t) = f(t, y(t)); & t \in I = [0,T] \\ y(0) = y_0 \end{cases} \tag{1}$$

where f is a continuous mapping from $R_+ \times R$ into R and $y_0 \in$ E with r- level sets

$[y_0]_r = [\underline{y}(0,r), \overline{y}(0,r)], r \in (0,1]$

The extension principle of Zadeh leads to the following definition of $f(t,y)$ when y = y(t) is a fuzzy number

$f(t,y)(s) = \sup\{y(\tau) | s = f(t,\tau)\}, s \in R$ It follows that

$[f(t,r)]_r = [\underline{f}(t,y,r), \overline{f}(t,y,r)], r \in (0,1],$ where

$$\left.\begin{aligned} \underline{f}(t,y,r) &= \min\{f(t,u) | u \in \underline{y}(r), \overline{y}(r),\} \\ \overline{f}(t,y,r) &= \max\{f(t,u) | u \in \underline{y}(r), \overline{y}(r),\} \end{aligned}\right\} \tag{2}$$

Theorem 1 [8]:

Let f satisfy $|f(t,v) - f(t,\overline{v})| \le g(t, |v - \overline{v}|), \ t \ge 0, \ v, \overline{v} \in R$ where $g : R_+ \times R_+$ is a continuous mapping such that $r \to g(t,r)$ is nondecreasing, the initial value problem $u'(t) = g(t,u(t))$ $u(0) = u_0$ (3) has a solution on R_+ for $u_0 > 0$ and that $u(t) = 0$ is the only solution of (3) for $u_0 = 0$. Then the fuzzy initial value problem (1) has a unique fuzzy solution.

4. Adomian Decomposition Method [2]:

Let us consider the second-order fuzzy ordinary differential equations of the form

$$\begin{cases} x''(t) = f(t, x, x') \\ x(t_0) = x_0, \quad x'(t_0) = x_0' \end{cases} \tag{4}$$

where x is a fuzzy function of t, $f(x,t)$ is a fuzzy function of the crisp variable t and the fuzzy derivative of x and $x(t_0)=x_0$ and $x'(t_0)=x_0'$ is a triangular shaped fuzzy number. We denote the fuzzy function x by $x=[\underline{x},\overline{x}]$. The α level set of $x(t)$ is defined as

$$[x(t)]_r=[\underline{x}(t,\alpha),\overline{x}(t,\alpha)] \text{ and } [x(t)]_r=[\underline{x}(t_0,\alpha),\overline{x}(t_0,\alpha)],\quad \alpha\in[0,1] \tag{5}$$

Consider the differential equation written in the following form

$$Lu+Ru+Nu=g \tag{6}$$

where L is linear operator, N is a non-linear operator, and $g(x)$ is a inhomogeneous term. where L is a linear operator, N is a nonlinear operator and $g(x)$ is an inhomogeneous term. If differential equations describes by n order, where the differential operator L is given by Adomian (1988). $L(.)=\frac{d^n(.)}{dt^n}$

the inverse operator L^{-1} is therefore considered a n-fold integral operator defined by $L^{-1}(*)=\int_0^x\int_0^x\ldots\int_0^x(*)dx\ldots dx,$ then

$$x=L^{-1}(*)(g(t)-Nx) \tag{7}$$

The Adomians technique consists of the solution of (6) as an infinite series

$$x=\sum_{n=0}^{\infty}x_n(t) \tag{8}$$

and decomposing the nonlinear operator N as

$$Nx=\sum_{n=0}^{\infty}A_n(t) \tag{9}$$

where An are Adomian polynomials of $x_0,x_1,x_2,\ldots,x_n$

$$A_n=\frac{1}{n}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i x_i\right)\right]_{\lambda=0} \tag{10}$$

n = 0, 1, 2 . . . substituting the derivatives (7), (8) and (9) which gives

$$\sum_{n=0}^{\infty}x_n(t)=L^{-1}(g(t))-L^{-1}\left(\sum_{n=0}^{\infty}A_n\right) \tag{11}$$

Thus $x_0=L^{-1}(g(t))$

$$x_{n+1}=L^{-1}\left(\sum_{n=0}^{\infty}A_n\right)=A_n(x_0,x_1,\ldots,x_n) \tag{12}$$

n = 0, 1, 2 ... We then define the k-term approximate to the solution x by

$$\Phi_k[x]=\sum_{n=0}^{\infty}x_n \quad and \quad \lim_{k\to\infty}\Phi_k[x]=x \tag{13}$$

Practical formula for the calculation of Adomian decomposition polynomials are given in A_n. However all term of the series cannot be determined usually $A_n=\sum_{n=0}^{\infty}x_n$ is approximated with truncated series of $\Phi_k=x_0+x_1+x_2\ldots+x_{n-1}$.

Lemma 1 [9]:

Let the sequence of numbers satisfy $\{W_n\}_{n=0}^{N}$, $|W_{n+1}|\le A|W_n|+B,\ 0\le n\le N-1$ for some given positive constants A and B . Then $|W_n|\le A^n|W_0|+B\frac{A^n-1}{A-1},\quad 0\le n\le N$

Lemma 2 [10]:

Let the sequence of numbers $\{W_n\}_{n=0}^{N}$, $\{V_n\}_{n=0}^{N}$ satisfy

$$\left.\begin{aligned}|W_{n+1}|&\le|W_n|+A\max\{|W_n|,|V_n|\}+B,\\|V_{n+1}|&\le|V_n|+A\max\{|W_n|,|V_n|\}+B.\end{aligned}\right\} \tag{14}$$

For some given positive constants A and B, and denote $U_n=|W_n|+|V_n|,0\le n\le N$

Then $U_n\le\bar{A}^nU_0+\bar{B}\frac{\bar{A}^{n-1}}{\bar{A}-1},\quad 0\le n\le N$ Where $\bar{A}=1+2A$ and $\bar{B}=2B$

Theorem 2 [11]:

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C^8(K)$ and let the partial derivatives of F and G be bounded over K. Then, for arbitrary fixed r, $0\le r\le 1$ the approximately solutions equation (12) converge to the exact solutions $\overline{y}(t,r)$ and $\underline{y}(t,r)$ uniformly in t.

5. Numerical Examples:

Example 1: Consider the fuzzy initial value problem

$$\begin{cases} y'(t) = y(t), t \in I = [0,1], \\ y(0) = (0.75 + 0.25r, 1.125 - 0.125r),\ 0<r \le 1 \end{cases} \quad (15)$$

The exact solution is given by

$\underline{y}(t,r) = (0.75 + 0.25r)e^t$ *and* $\overline{y}(t,r) = (1.125 - 0.125r)e^t, 0<r \le 1$

Applying equation (15) the inverse operator L^{-1} we obtain

$\overline{y}(t,r) = (1.125 - 0.125r) + L^{-1}(\overline{y}(t,r))$

Using the decomposition series of $\overline{y}(t,r)$ we get

$$\sum_{n=0}^{\infty} \overline{y}_n(t,r) = (1.125 - 0.125r) + \int_0^t \overline{y}_{n-1}(t,r)dt$$

$\overline{y}_0(t,r) = (1.125 - 0.125r)$, $\overline{y}_1(t,r) = (1.125 - 0.125r)t$, $\overline{y}_2(t,r) = (1.125 - 0.125r)\frac{t^2}{2!}$

$$\overline{y}_n(t,r) = (1.125 - 0.125r)\left(1 + t + \frac{t^2}{2!} + \ldots\right) \quad (16)$$

and

$$\underline{y}_n(t,r) = (0.75 + 0.25r)\left(1 + t + \frac{t^2}{2!} + \ldots\right) \quad (17)$$

Equations (16) and (17) provides approximate solution of $\underline{y}_n(t,r)$ and $\overline{y}_n(t,r)$ in example 1 which is same as exact solution . The computational results are presented in Table 1.The approximate solution of $\underline{y}_n(t,r)$ and $\overline{y}_n(t,r)$ are plotted in figure 1 and figure 3. The graph of error functions $y(t,r)$ - $y_n(t,r)$ are shown in figure 2.

Table 1: Comparing numerical values of $\underline{y}(t,r)$ and $\overline{y}(t,r)$ using Runge-Kutta method [8] and Fuzzy Adomian decomposition method with Exact solution for different values of r at $t=1$.

r	R-K Method [8]		Exact solution [6]		Present method	
	$\underline{y}(t,r)$	$\overline{y}(t,r)$	$\underline{y}(t,r)$	$\overline{y}(t,r)$	$\underline{y}(t,r)$	$\overline{y}(t,r)$
0	2.0388	3.0242	2.0387	3.0581	2.0313	3.0469
0.1	2.1066	3.0238	2.1067	3.0241	2.0991	3.0131
0.2	2.1064	2.9898	2.1746	2.9901	2.1667	2.9792
0.3	2.1744	2.9559	2.2426	2.9561	2.2344	2.9453
0.4	2.2424	2.9219	2.3105	2.9222	2.3021	2.9115
0.5	2.3104	2.8879	2.3785	2.8882	2.3698	2.8776
0.6	2.3783	2.8539	2.4465	2.8542	2.4375	2.8438
0.7	2.4463	2.8199	2.5144	2.8202	2.5052	2.8099
0.8	2.5442	2.7861	2.5824	2.7862	2.5729	2.7761
0.9	2.6501	2.7521	2.6503	2.7523	2.6406	2.7422
1	2.7182	2.7181	2.7183	2.7183	2.7083	2.7083

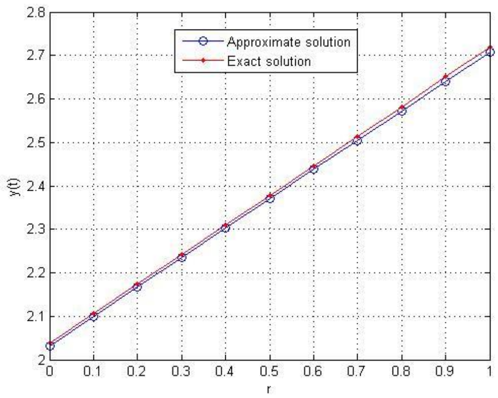

Figure 1: The approximate solution of $\overline{y}(t,r)$ for various values of r when t= 1with exact solution

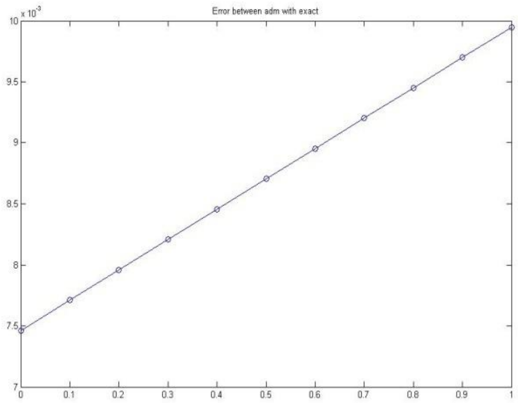

Figure 2: Error between Adomian decomposition method and Exact for $\overline{y}(t,r)$

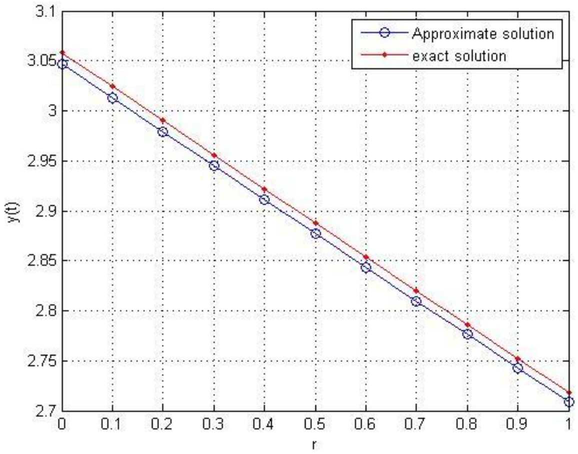

Figure 3: Numerical solution of $\underline{y}(t,r)$ for various values of *r* when t= 1

Example 2 [8]: Consider the fuzzy intial value problem,

$y'(t) = c_1 y^2(t)+c_2;\ y(0) = 0$ where $c_i > 0$,for $i = 1,2$ are triangular fuzzy numbers (18)

The exact solution is given by $y(t, r) = l_1(r)\tan(w_1(r)t)$ and $y_2(t, r) = l_2(r)\tan(w_2(r)t)$

$$l_1(r) = \sqrt{\frac{c_{2,1}(r)}{c_{1,1}(r)}} \qquad l_2(r) = \sqrt{\frac{c_{2,2}(r)}{c_{1,2}(r)}}$$

$$w_1(r) = \sqrt{\frac{c_{1,1}(r)}{c_{2,1}(r)}} \qquad w_2(r) = \sqrt{\frac{c_{1,2}(r)}{c_{2,2}(r)}}$$

$[c_1]_r = [0.5+0.5r, 1.5-0.5r]$ and $[c_2]_r = [0.75+0.25r, 1.25-0.25r]$

The *r*-level sets of $y'(t)$ are

$$y_1'(t,r) = c_{2,1}(r)\sec^2(w_1(r)t)$$

$$y_2'(t,r) = c_{2,2}(r)\sec^2(w_2(r)t)$$

which defines a fuzzy number. We have

$$f_1(t,y,r) = \min\left\{c_1u^2 + c_2 \middle| u \in [y_1(t,r), y_2(t,r)], c_1 \in [c_{1,1}(r), c_{1,2}(r)], c_2 \in [c_{2,1}(r), c_{2,2}(r)]\right\}$$

$$f_2(t,y,r) = \max\left\{c_1u^2 + c_2 \middle| u \in [y_1(t,r), y_2(t,r)], c_1 \in [c_{1,1}(r), c_{1,2}(r)], c_2 \in [c_{2,1}(r), c_{2,2}(r)]\right\}$$

Apply decomposition method we have

$$Ly'(t) = c_1 Ly(t)^2 + Lc_2 \qquad (19)$$

Apply L^{-1} on both side we get

$$y(t) - y(0) = c_1\int_0^t y(t)^2\,dt + c_2\int_0^t dt \qquad (20)$$

$$y(t) = c_1 \int_0^t y(t)^2\, dt + c_2 t \tag{21}$$

Equation (21) can be decomposed into

$$y_0(t) = c_2 t$$

$$y_n(t) = c_1 \int_0^t y_{n-1}(t)^2\, dt \tag{22}$$

$$y_1(t) = \frac{c_1 c_2^2 t^3}{3}$$

$$y_2(t) = \frac{2 c_1^2 c_2^3 t^5}{15}$$

...

$$y_n(t) = c_2 t + \frac{c_1 c_2^2 t^3}{3} + \frac{2 c_1^2 c_2^3 t^5}{15} + \frac{17 c_1^3 c_2^4 t^7}{315} + \ldots.. \tag{23}$$

The *r*-level sets of $y_n(t)$ can be written as

$$\overline{y}_n(t,r) = \overline{c}_2 t + \frac{\overline{c}_1 \overline{c}_2^2 t^3}{3} + \frac{2 \overline{c}_1^2 \overline{c}_2^3 t^5}{15} + \frac{17 \overline{c}_1^3 \overline{c}_2^4 t^7}{315} + \ldots.. \tag{24}$$

and

$$\underline{y}_n(t,r) = \underline{c}_2 t + \frac{\underline{c}_1 \underline{c}_2^2 t^3}{3} + \frac{2 \underline{c}_1^2 \underline{c}_2^3 t^5}{15} + \frac{17 \underline{c}_1^3 \underline{c}_2^4 t^7}{315} + \ldots.. \tag{25}$$

Equations (24) and (25) provides approximate solution of $\underline{y}_n(t,r)$ and $\overline{y}_n(t,r)$ for n=7 using MATLAB programme . The computational results are presented in the below figures.

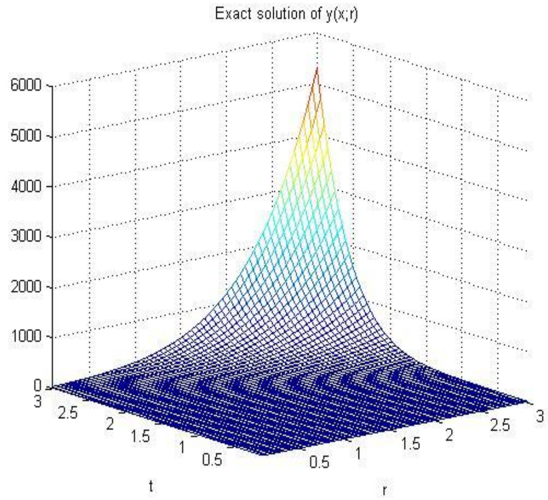

Figure 4: Approximate and Exact solution of $\overline{y}(t,r)$ for various values of *r* when t= 1

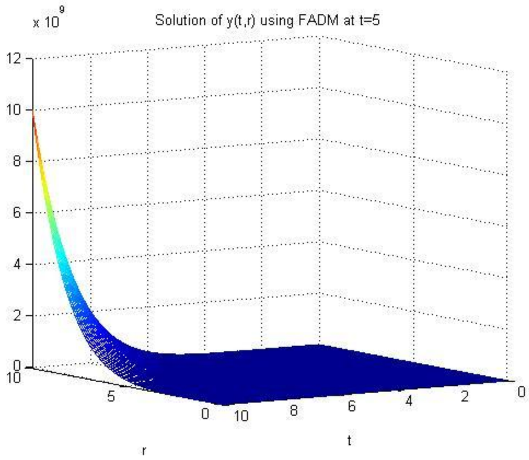

Figure 5: Approximate of $\overline{y}(t,r)$ for various values of *r* when t= 5 using FADM

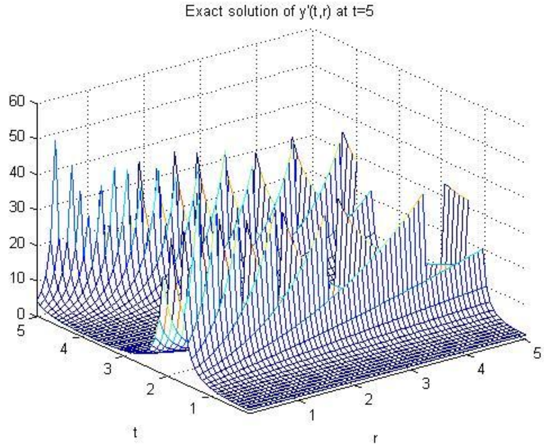

Figure 6: Approximate and Exact solution of $\overline{y}'(t,r)$ for various values of *r* when t= 5

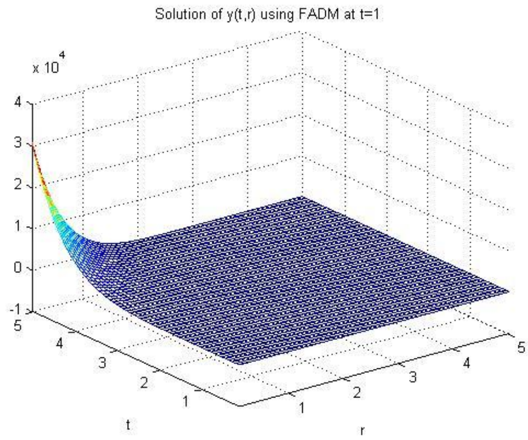

Figure 7: Approximate solution of $\underline{y}(t,r)$ for various values of *r* when t= 1using FADM

Example 4.3: Consider the following nonlinear fuzzy differential equation with fuzzy initial value is given [12].

$$y'(t) = t^2 y(t) - 4ty(t) + 3y(t), t \in [0,2] \qquad (26)$$
$$y(0) = (0,1,1)$$

Applying equation (26) the inverse operator L^{-1} we obtain

$$y_n(t) = 1 + \int_0^t t^2 y_{n-1}(t) - 4ty_{n-1}(t) + 3y_{n-1}(t)$$

Using the decomposition series of *y(t)* we get

$$y(t) = 1 + 4t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{12} + ... \qquad (27)$$

Equation (27) is approximate solution of y(t).

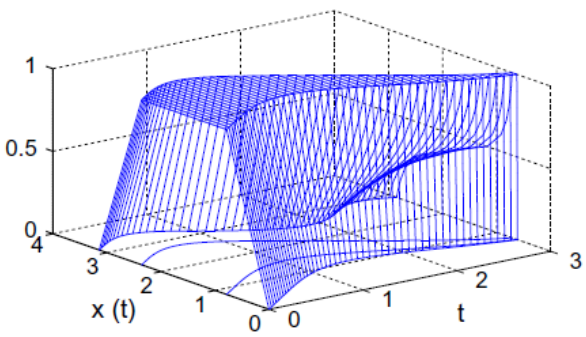

Figure 8: Solution Graph for y(t) using Fuzzy Adomian decomposition Method

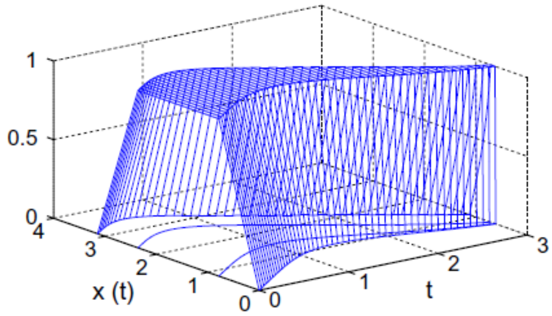

Figure 9: Exact solution for y(t) in Example 4.3

6 Conclusion:

The main destination of this paper has been to derive an analytical parametric solution for the fuzzy Cauchy differential equation. We have achieved this purpose by applying Adomian decomposition method.This method has a useful feature in that it provides the solution in a rapid convergent power series with elegantly computable convergence of the solution.

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